Introduction to Incremental View Maintenance

Dan Olteanu (University of Zurich)

fdbresearch.github.io

Berkeley CS 294-248: Topics in Database Theory

https://berkeley-cs294-248.github.io/

September 12 & 14, 2023

Short Biography

Since 2020: Professor of Computer Science, University of Zurich

Since 2017: Computer Scientist at RelationalAI

2007 - 2020: Professor of Computer Science, University of Oxford

2013 - 2014: Visiting professor at University of California, Berkeley

Taught CS 186 & worked for LogicBlox

Acknowledgments

Members of the DaST IVM team

Ahmet Kara

Milos Nikolic

Haozhe Zhang

Real-Time Analytics

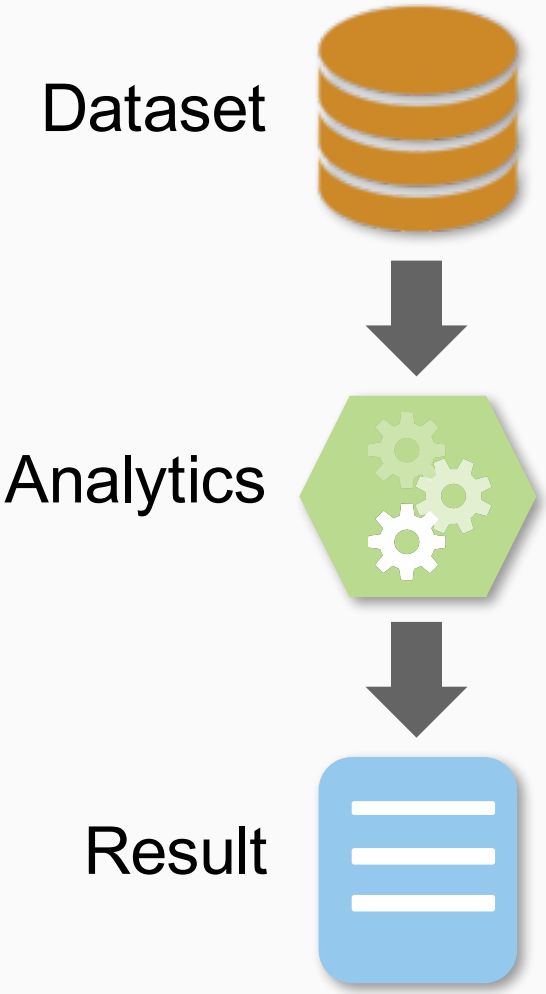
- Datasets continuously evolve over time
  - E.g.: data streams from sensors, social networks, apps
- Real-time analytics over streaming data
  - Users want fresh up-to-date computation results, e.g., models

Web Analytics

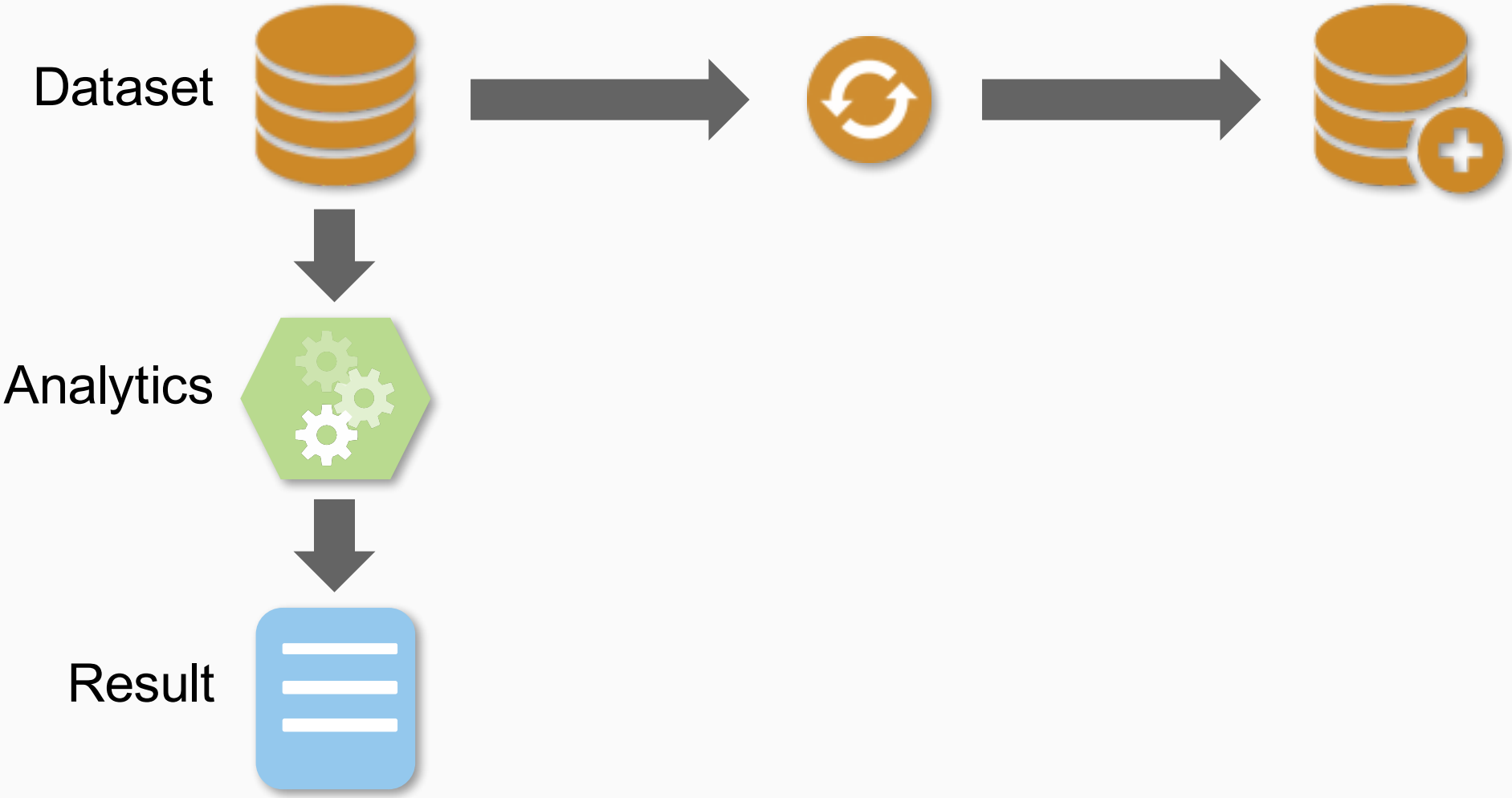
Sensor Networks

Cloud Monitoring

Real-Time Analytics via Incremental Maintenance



Real-Time Analytics via Incremental Maintenance



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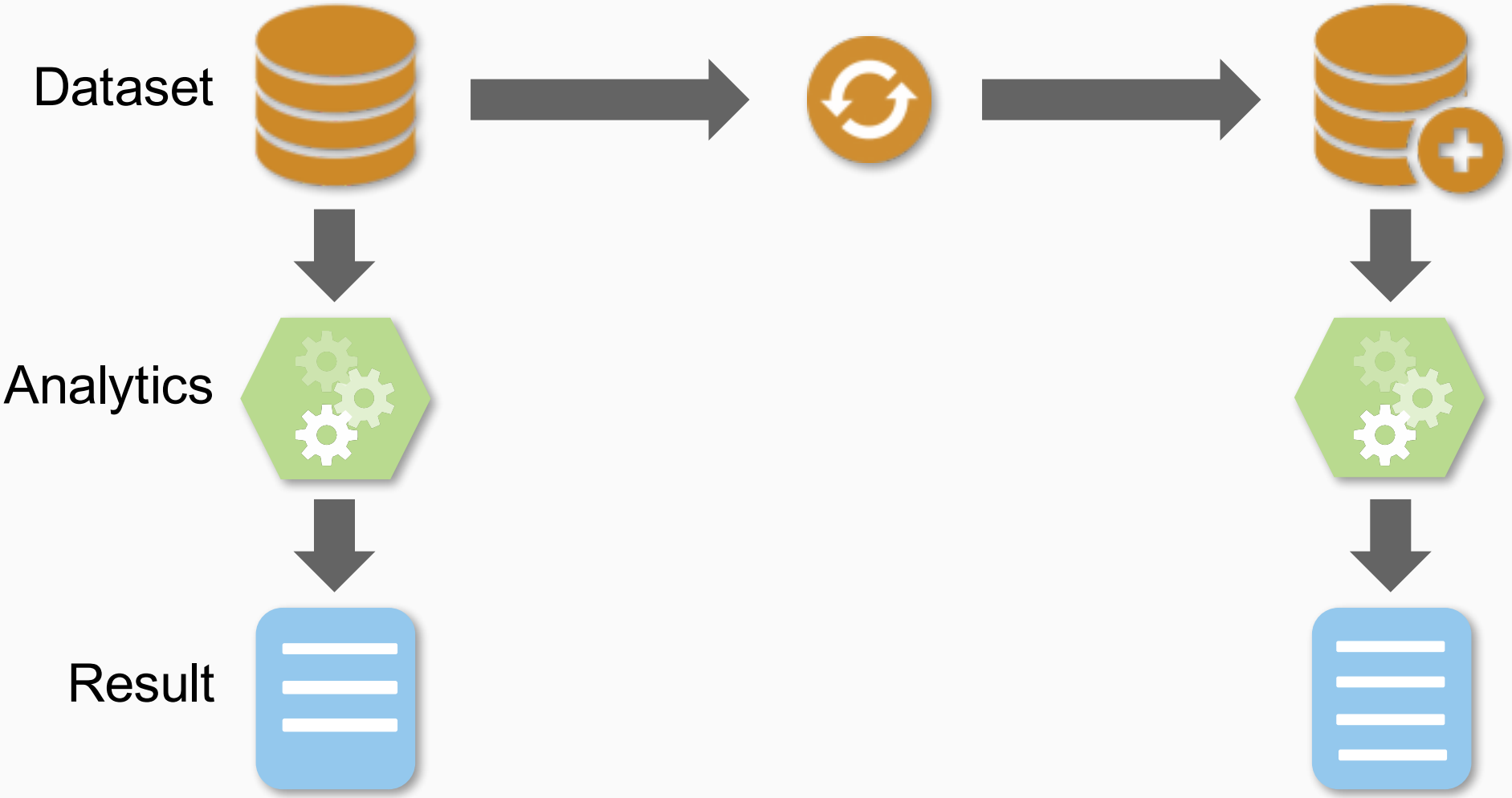

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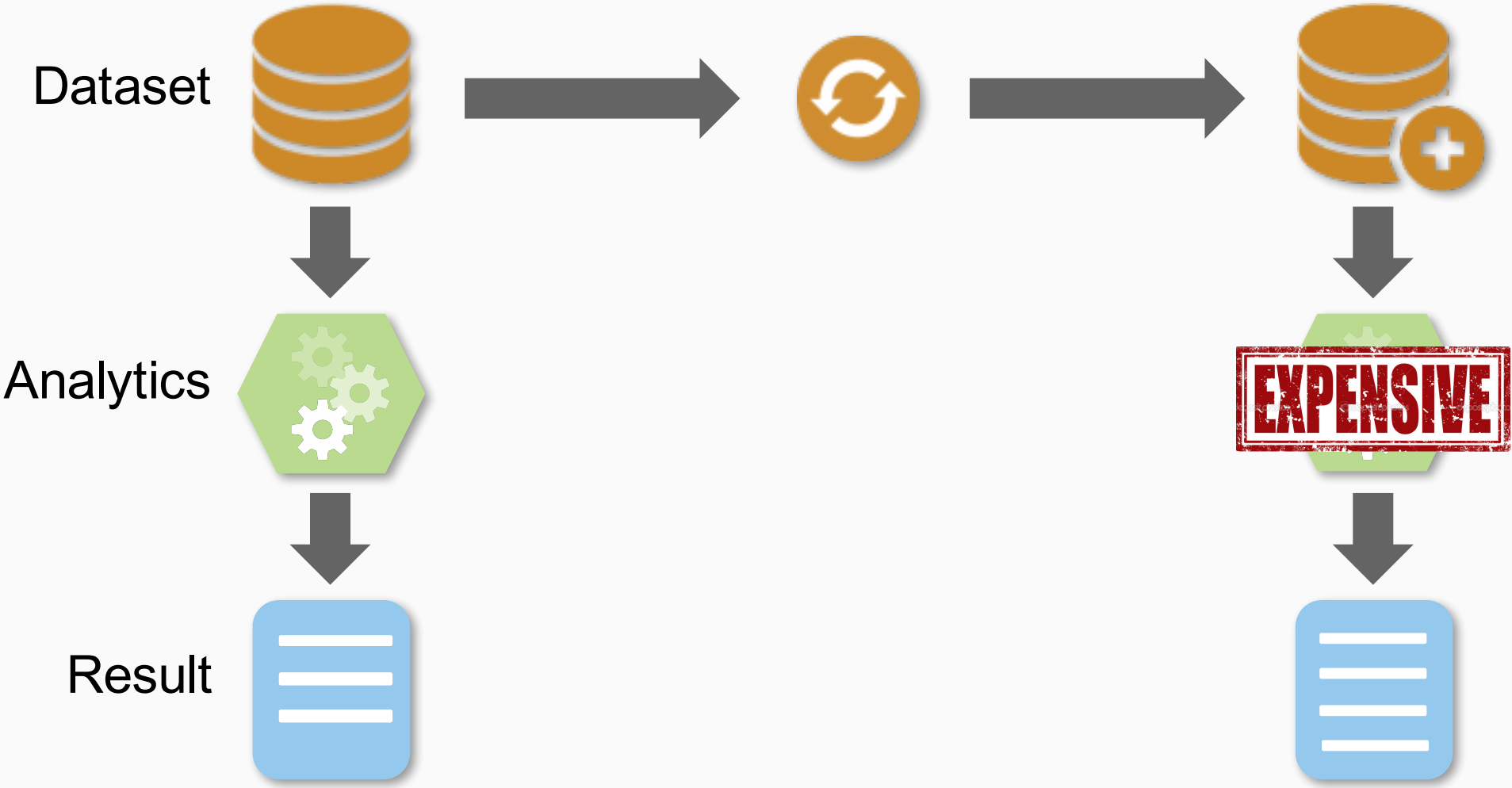

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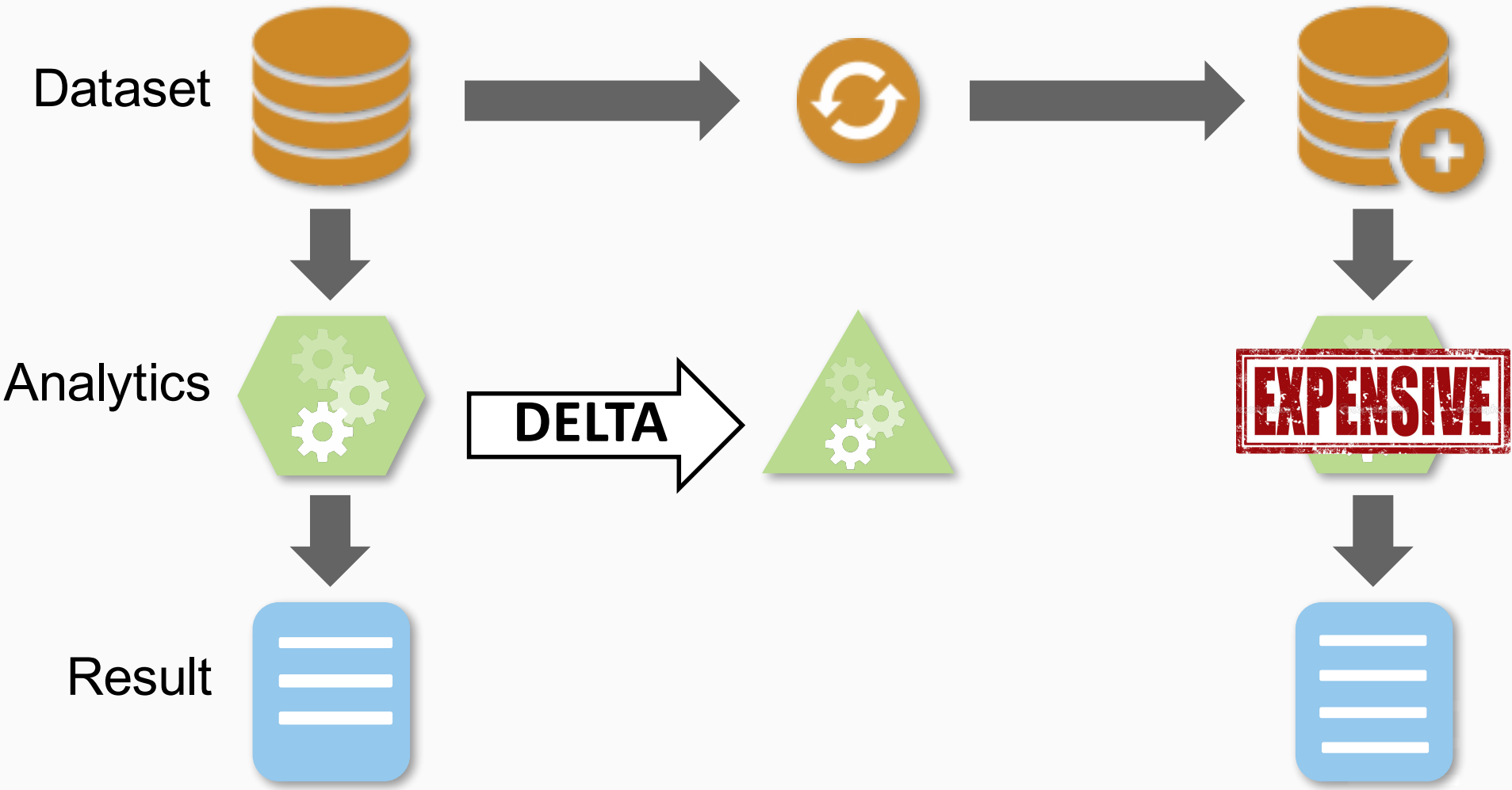

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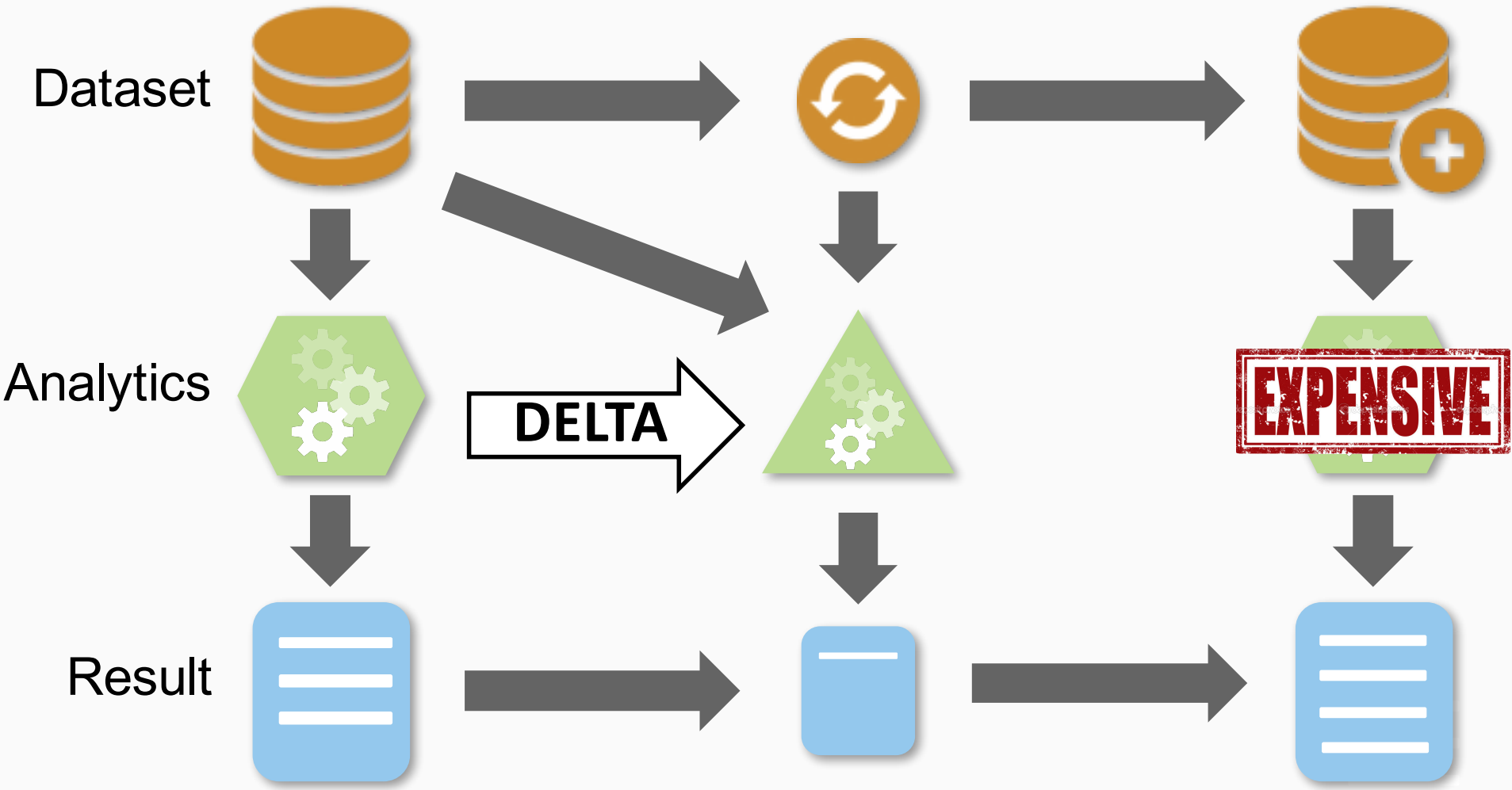

Setting & Objective of this Lecture

Incremental View Maintenance (IVM)

- Well-established and longstanding research problem
- Confusing naming: incremental vs decremental

  Alternative common naming: *Fully dynamic*

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- Single-tuple updates to relational databases
- Relational queries (non-recursive)

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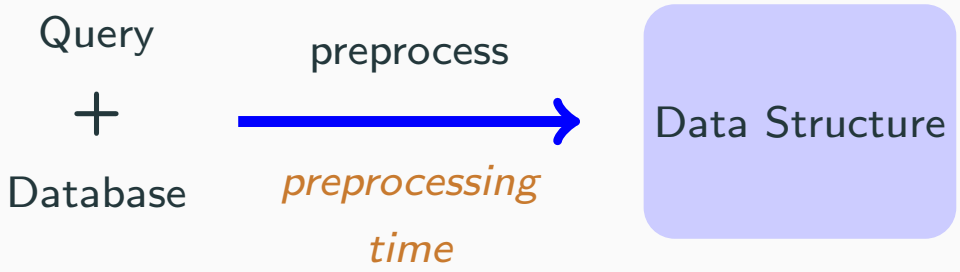
Objectives

- Main IVM approaches: First/higher order, adaptive
- Which queries can be maintained optimally?

The Incremental View Maintenance Problem



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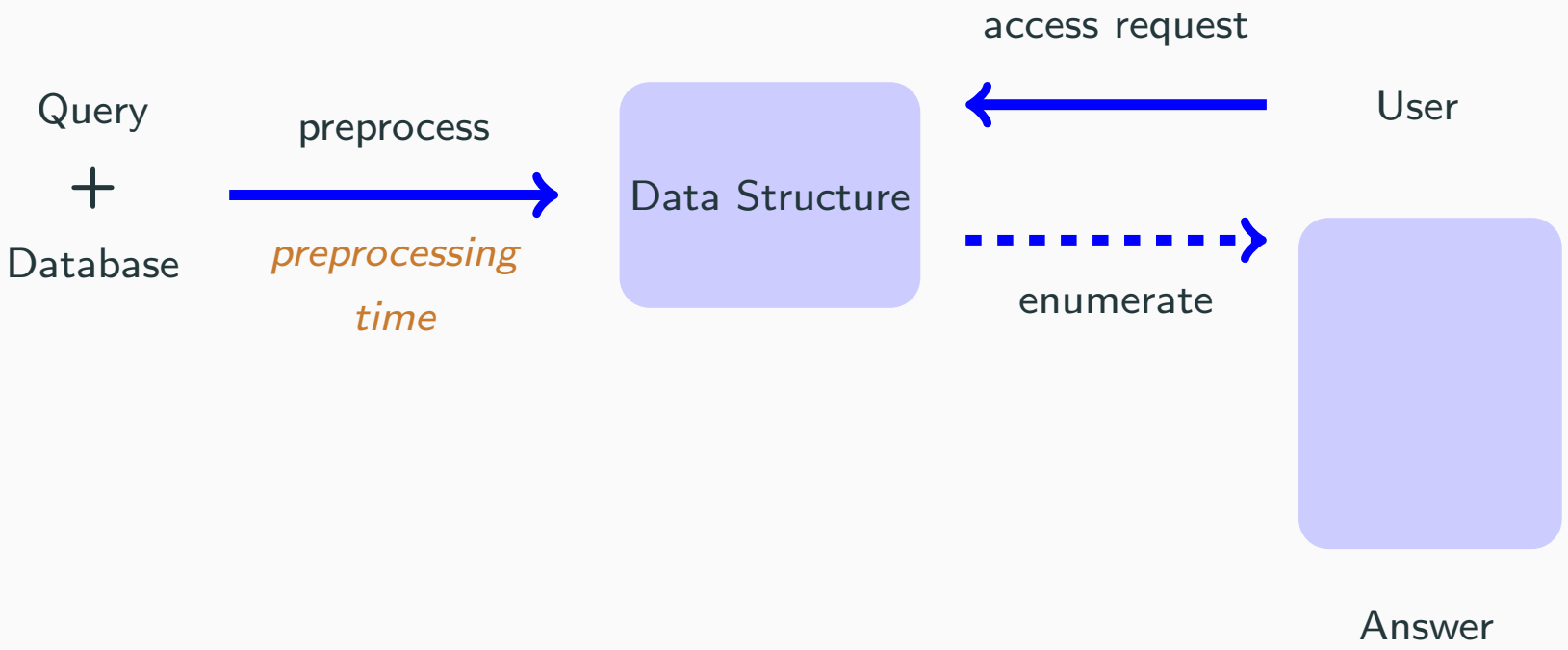

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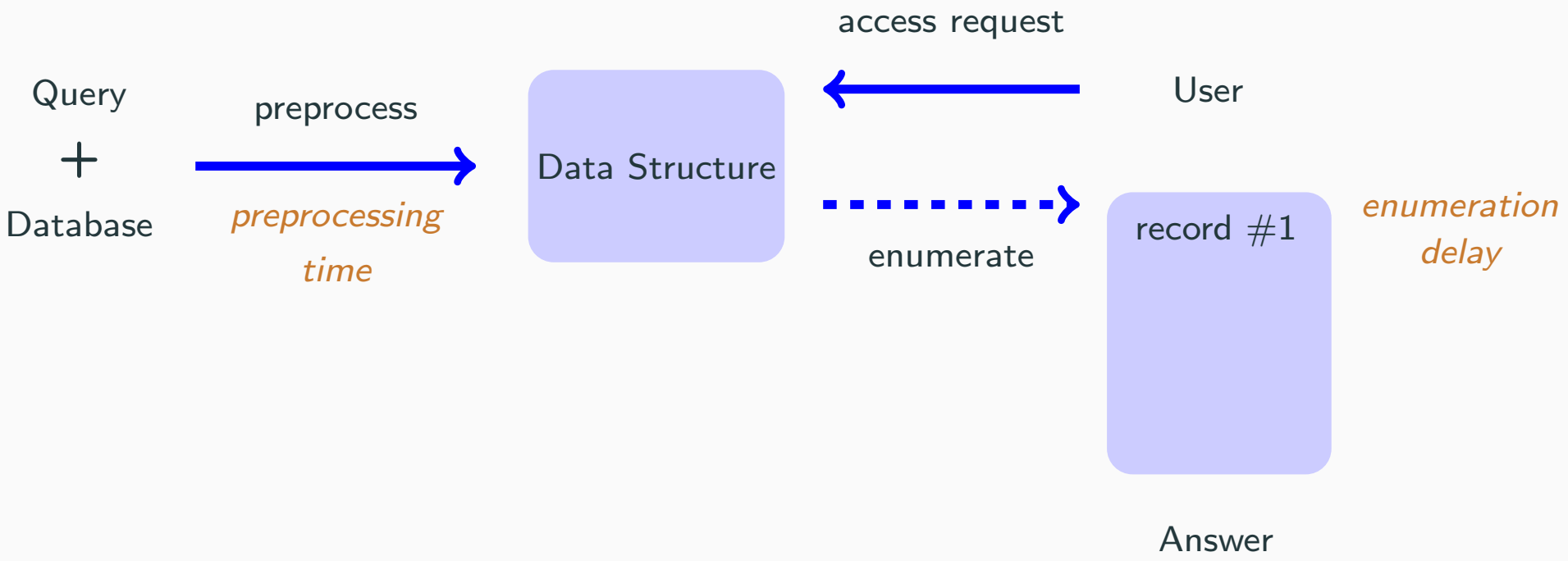

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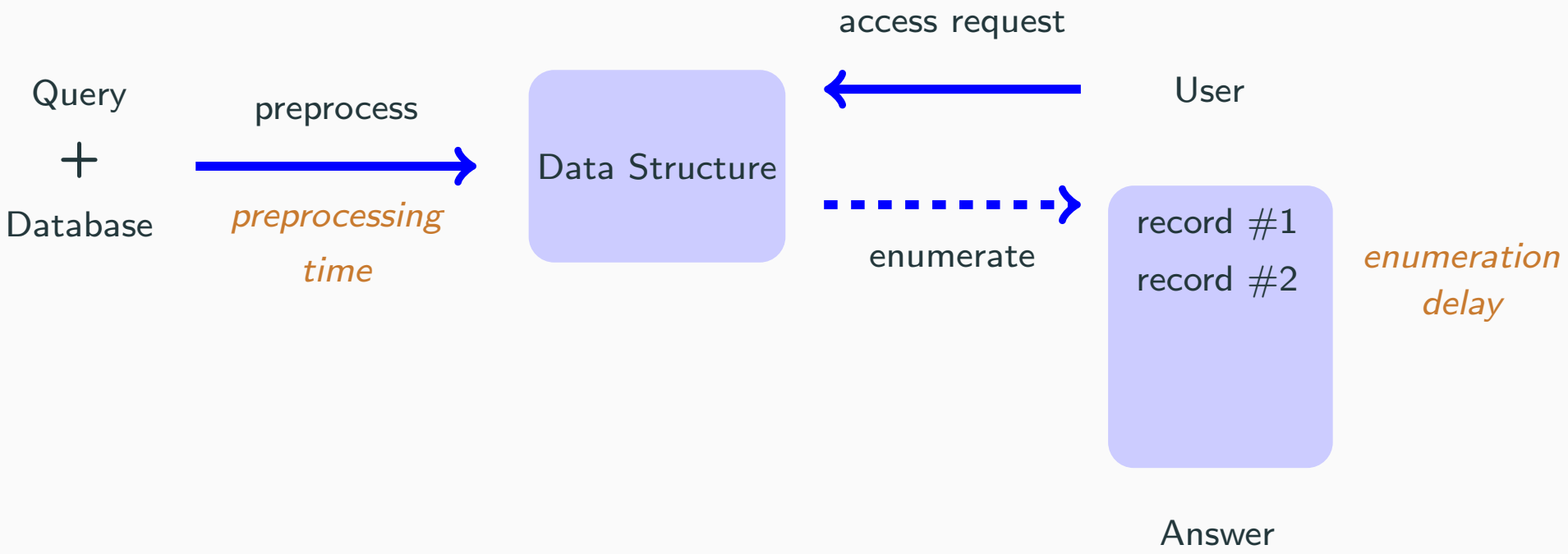

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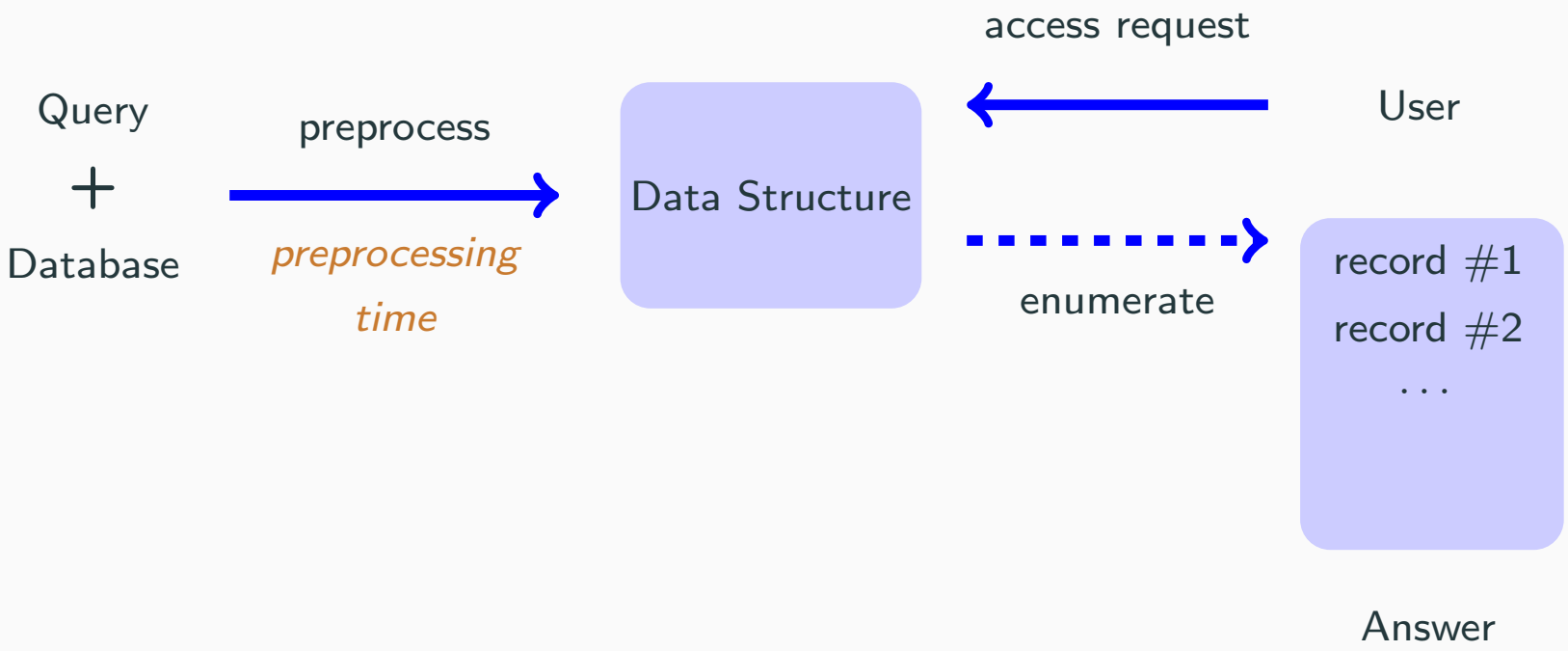

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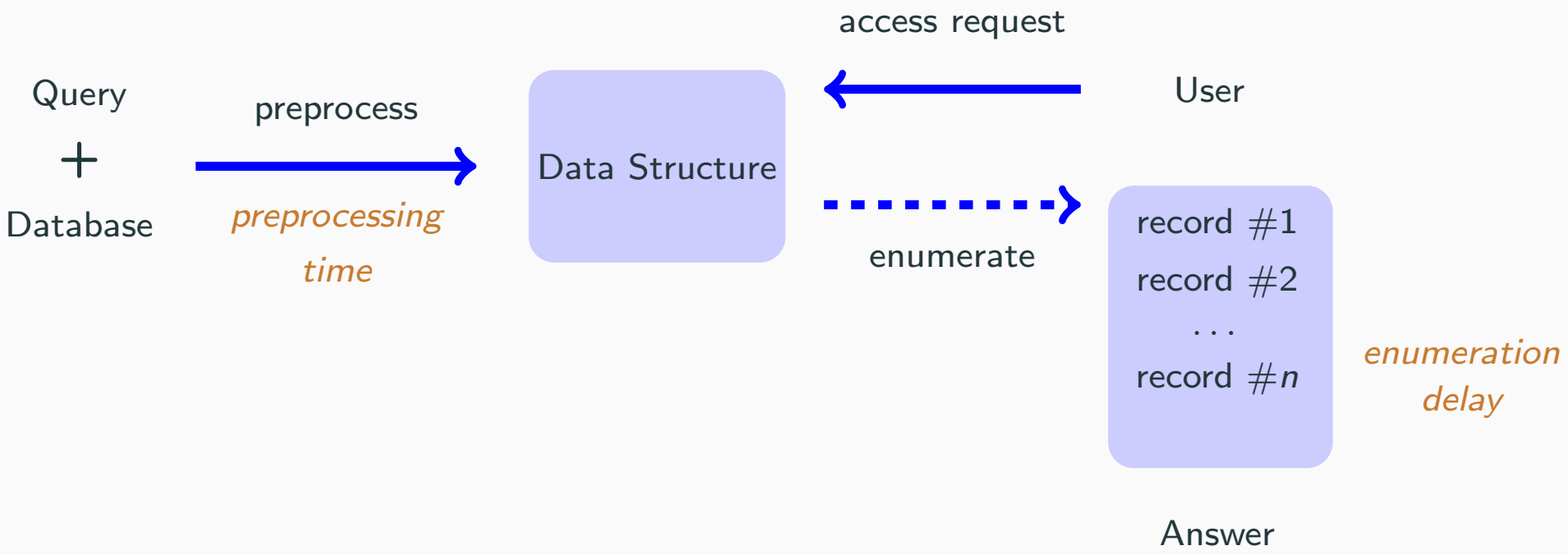

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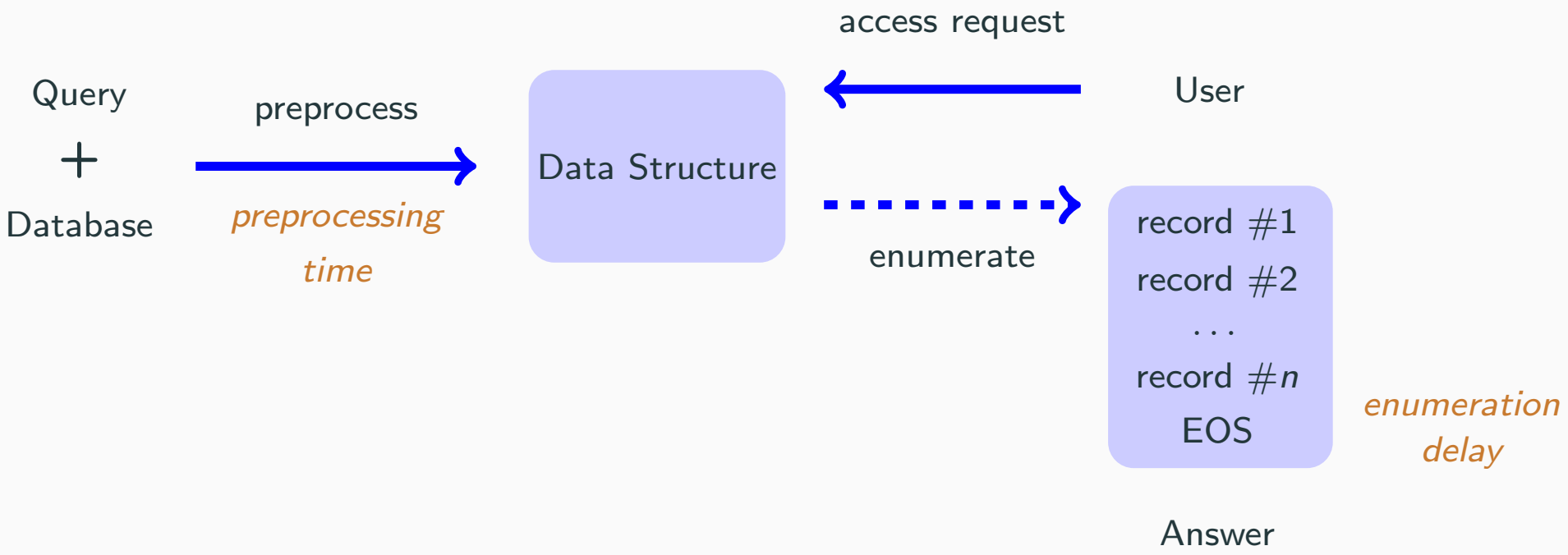

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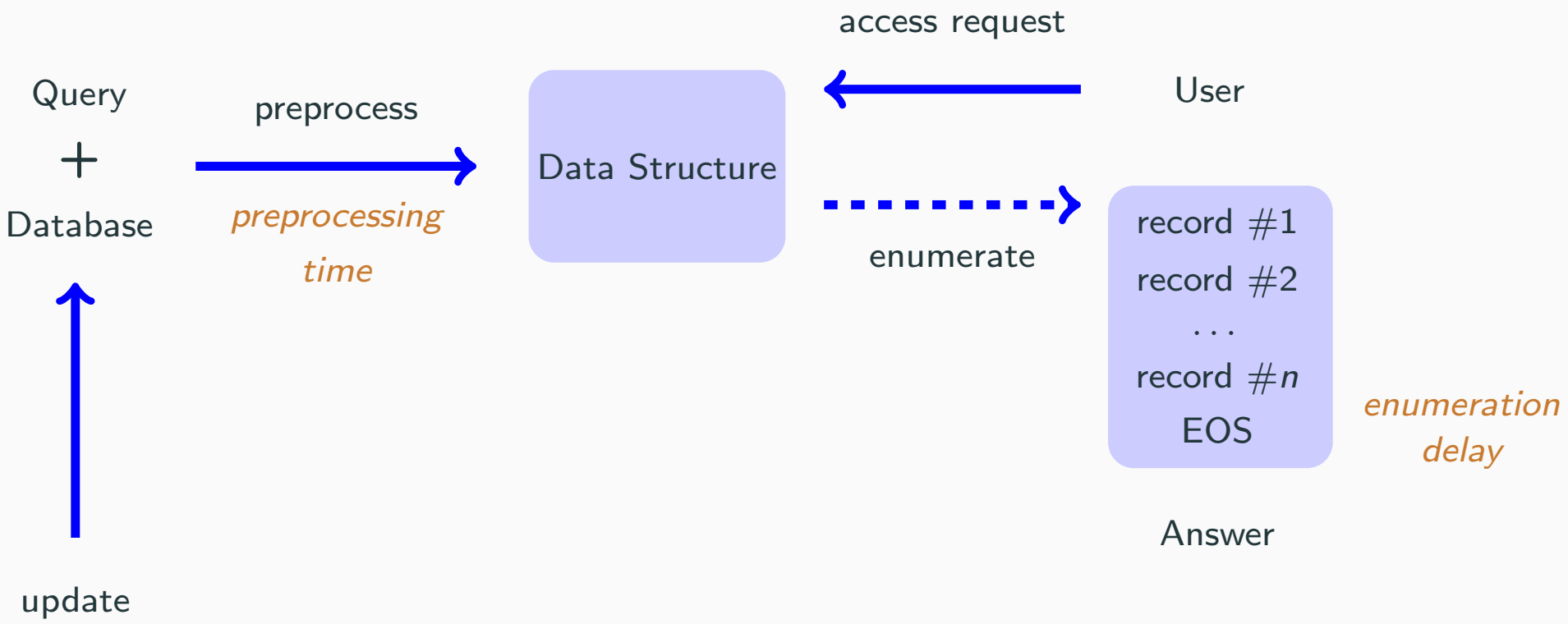

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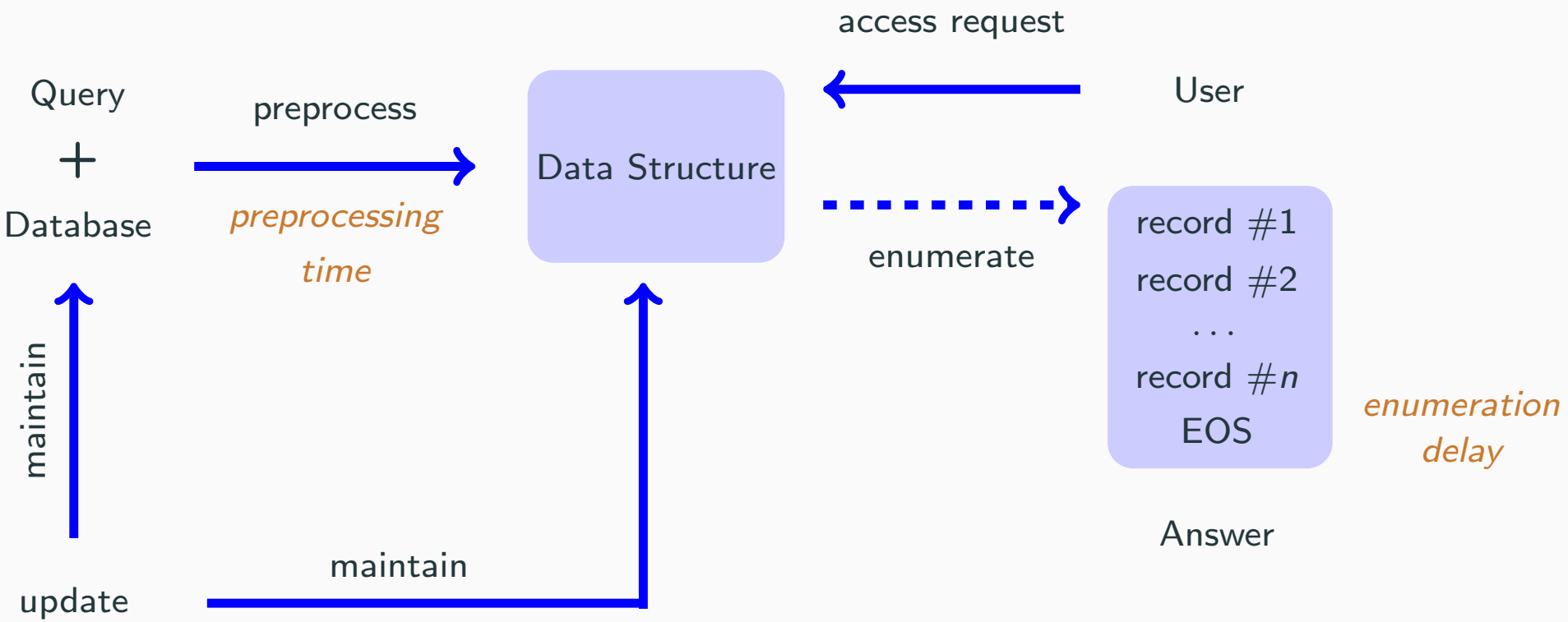

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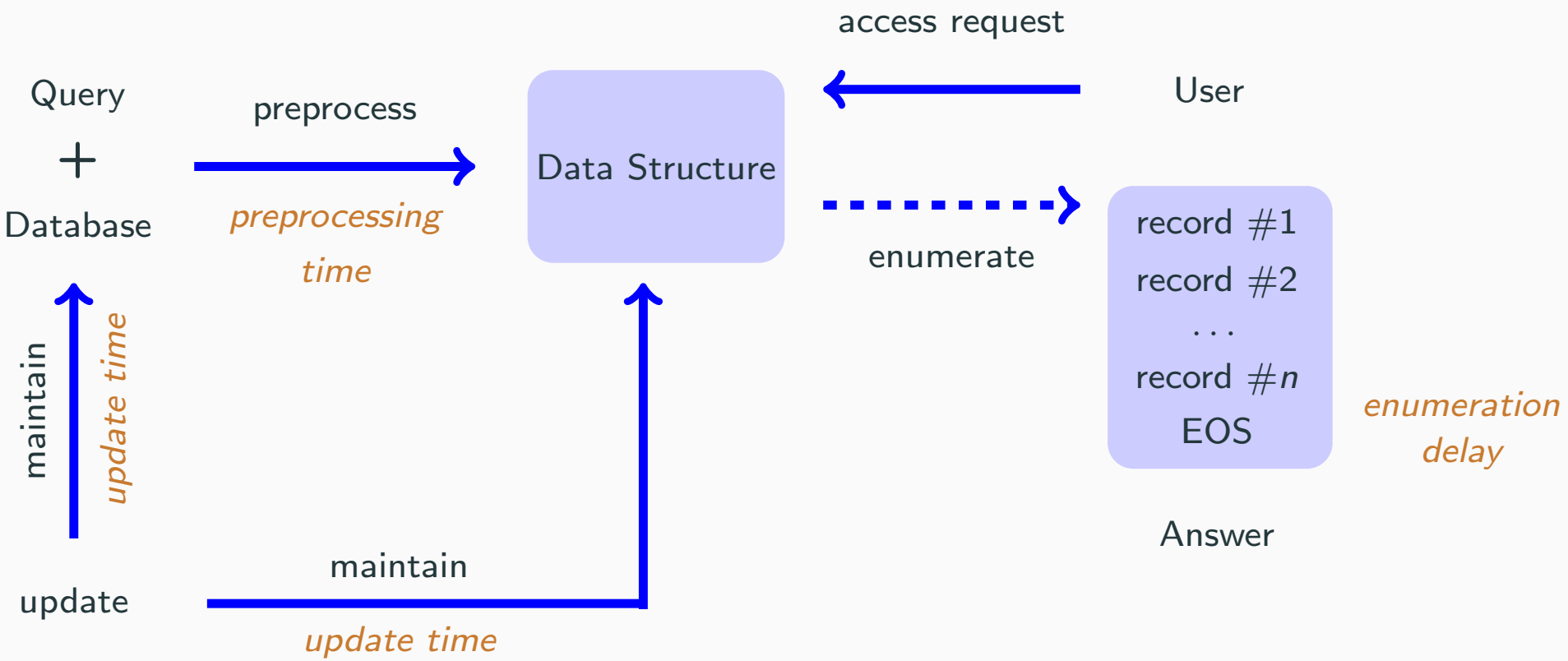


Agenda

Part 1. Data model and query language

Part 2. Main IVM techniques by example

- First-order IVM using delta queries
- Higher-order IVM using DBToaster and F-IVM
- Adaptive IVM using IVM$^\epsilon$

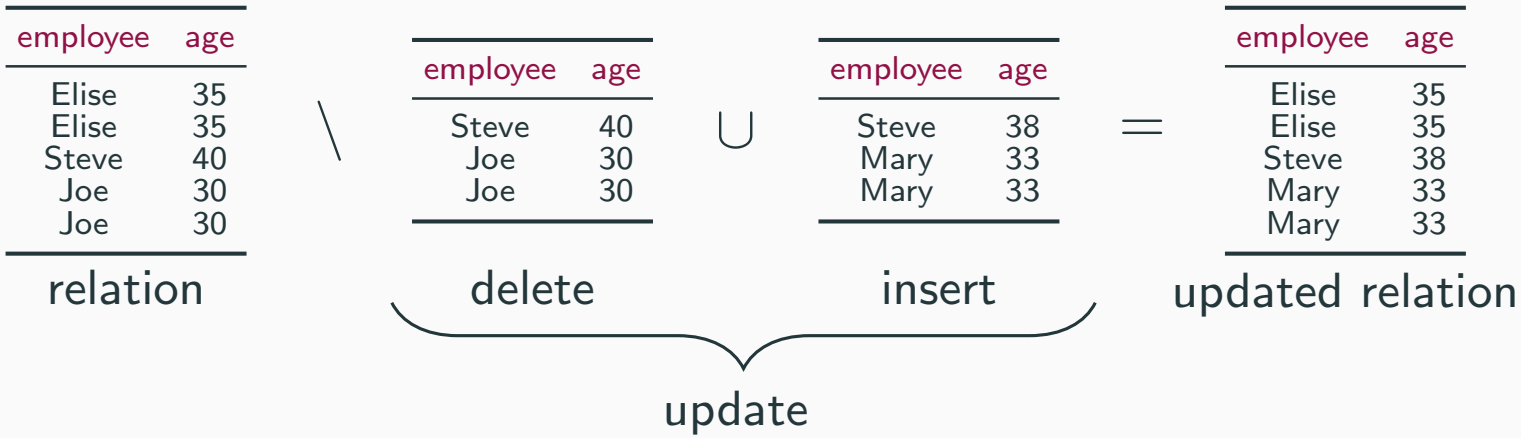
Part 3. Which queries can be maintained optimally?

- Constant update time and enumeration delay
- Beyond constant time

Part 1.
Data Model &
Query Language

Classical Representation of Relations and Updates

Insertions and deletions are represented as separate tables:



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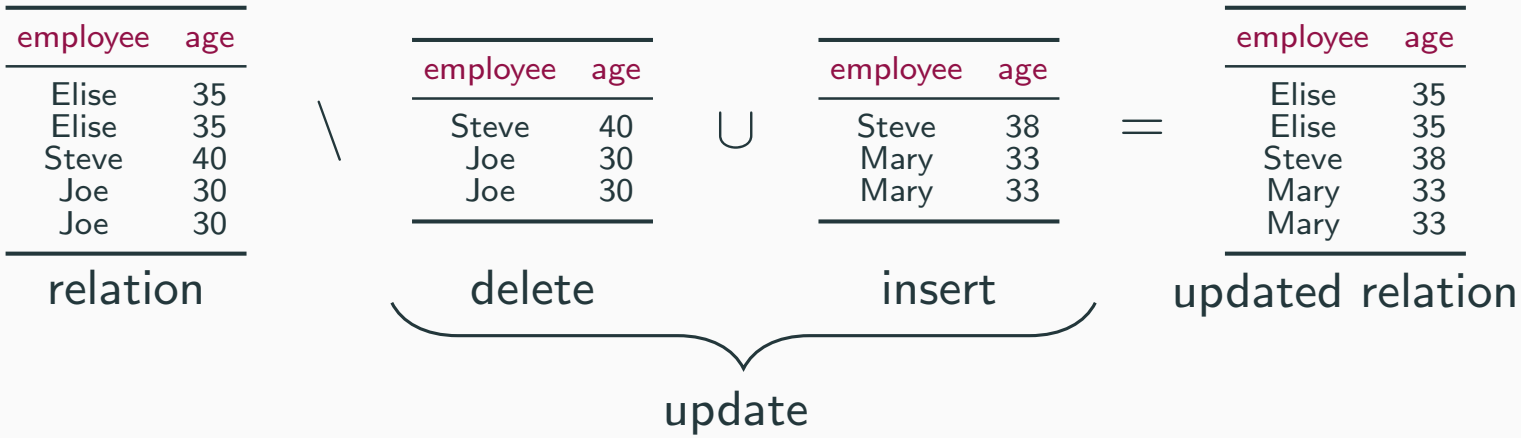

- Order of updates matters!

  This affects the parallel and distributed execution of updates

- Inserts and deletes are treated differently

  They rely on different execution mechanisms

Uniform Representation of Relations and Updates

Relations and updates are represented as factors mapping tuples to multiplicities:

employee	age	→	#
Elise	35	→	2
Steve	40	→	1
Joe	30	→	2

factor

+

employee	age	→	#
Steve	40	→	−1
Steve	38	→	1
Joe	30	→	−2
Mary	33	→	2

update

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- Order of updates does not matter: Addition is associative
- Database may be in inconsistent state: tuples with negative multiplicities
- Multiplicity:

  negative = tuple delete; positive = tuple insert; 0 = tuple not in database

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From $\mathbb{Z}$ to arbitrary rings (r_1 and r_2 are elements from a ring):

A	B	→	R(A, B)
a_1	b_1	→	r_1
a_2	b_1	→	r_2

Rings

- A ring $(\mathbf{D}, +, \cdot, \mathbf{0}, \mathbf{1})$ is a set $\mathbf{D}$ with two binary ops:

Additive commutativity	$a + b = b + a$
Additive associativity	$(a + b) + c = a + (b + c)$
Additive identity	$\mathbf{0} + a = a + \mathbf{0} = a$
Additive inverse	$\exists - a \in \mathbf{D} : a + (-a) = (-a) + a = \mathbf{0}$
Multiplicative associativity	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Multiplicative identity	$a \cdot \mathbf{1} = \mathbf{1} \cdot a = a$
Left and right distributivity	$a \cdot (b + c) = a \cdot b + a \cdot c$ and $(a + b) \cdot c = a \cdot c + b \cdot c$

  The ring is commutative if multiplication is also commutative

- Examples: $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$
- More on semi-rings later in the course
- We use the ring $(\mathbb{Z}, +, \cdot, 0, 1)$

Operations on Factors

R(a, b)

A	B	→	#
a_1	b_1	→	r_1
a_2	b_1	→	r_2

S(a, b)

A	B	→	#
a_2	b_1	→	s_1
a_3	b_2	→	s_2

T(b, c)

B	C	→	#
b_1	c_1	→	t_1
b_2	c_2	→	t_2

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Union/Sum V(a, b) = R(a, b) + S(a, b)

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a_1	b_1	→	r_1
a_2	b_1	→	$r_2 + s_1$
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Join/Product

$W(a, b, c) = V(a, b) \cdot T(b, c)$

A	B	C	→	#
a_1	b_1	c_1	→	$r_1 \cdot t_1$
a_2	b_1	c_1	→	$(r_2 + s_1) \cdot t_1$
a_3	b_2	c_2	→	$s_2 \cdot t_2$

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Project/Marginalization

$U(b, c) = \sum_a W(a, b, c)$

B	C	→	#
b_1	c_1	→	$r_1 * t_1 + (r_2 + s_1) \cdot t_1$
b_2	c_2	→	$s_2 \cdot t_2$

Query Language Considered in This Lecture

Language of queries with operations sum, product, and variable marginalization over factors. Examples:

- List the paths of length two in a graph with edge factor E:

$$Q(a, b, c) = E(a, b) \cdot E(b, c)$$

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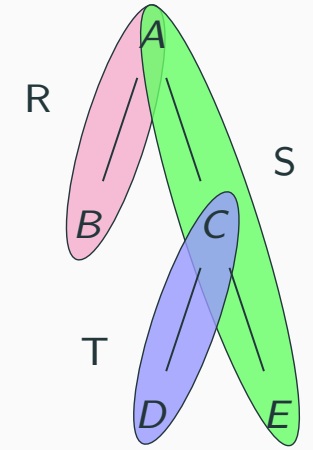
- Update a factor E with changes given by another factor δE:

$$E_{new}(a, b) = E(a, b) + \delta E(a, b)$$

Query Evaluation Example (1/2)

Compute COUNT over the natural join of factors R, S, and T (using Yannakakis)

Factors map tuples to multiplicity 1



Join hypergraph

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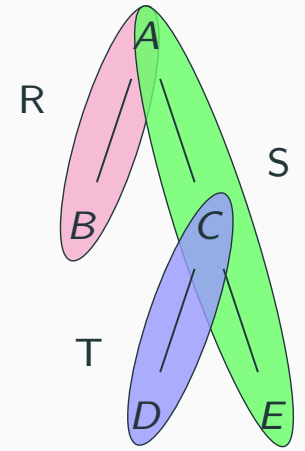

Join hypergraph

Naïve: compute the join and then COUNT

$$\mathrm{Q} = \sum_{a,b,c,d,e} \mathrm{R}(\mathrm{a},\mathrm{b}) \cdot \mathrm{S}(\mathrm{a},\mathrm{c},\mathrm{e}) \cdot \mathrm{T}(\mathrm{c},\mathrm{d})$$

Takes $\mathcal{O}(N^3)$ time if each factor has size $\mathcal{O}(N)$!

There can be $\mathcal{O}(N^3)$ possible combinations of b, e, and d values

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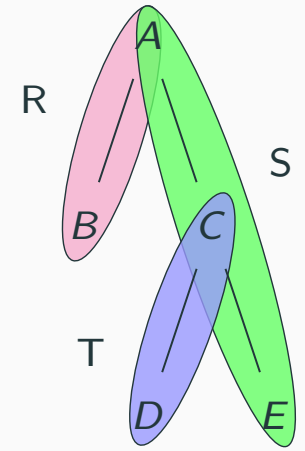

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Naïve: compute the join and then COUNT

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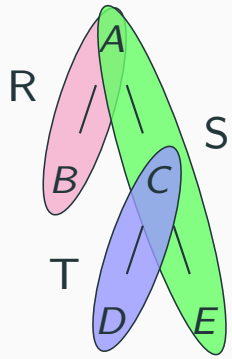
Can we compute COUNT in $\mathcal{O}(N)$ time?

Query Evaluation Example (2/2)

Idea: Push COUNT past joins and marginalize variables



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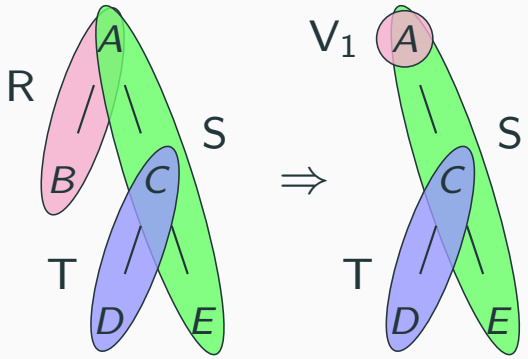

$\Rightarrow$

$$V_1(a) = \sum_b R(a, b) \qquad \mathcal{O}(N)$$

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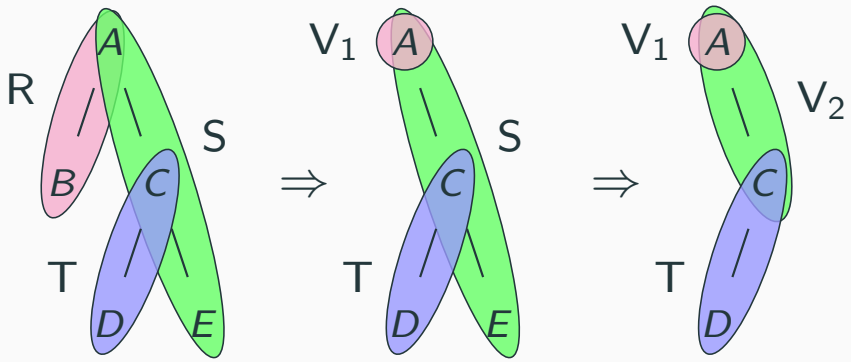
Idea: Push COUNT past joins and marginalize variables



$$V_1(a) = \sum_b R(a, b) \qquad \mathcal{O}(N)$$

$$V_2(a, c) = \sum_e S(a, c, e) \qquad \mathcal{O}(N)$$

Query Evaluation Example (2/2)

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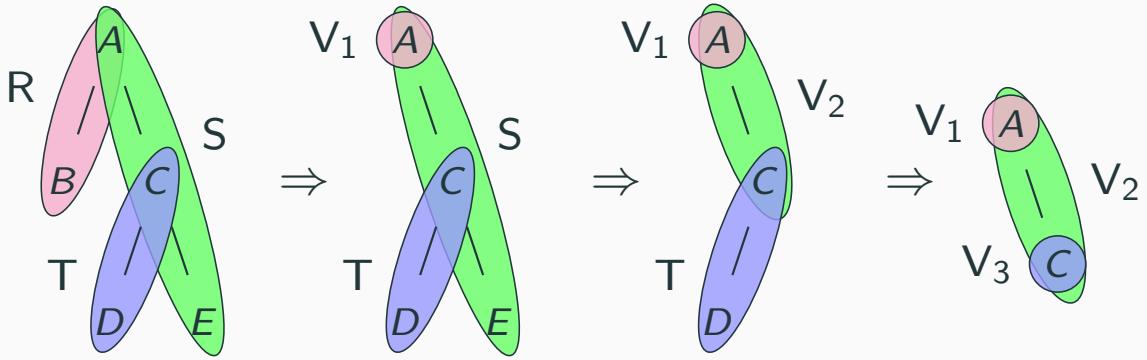

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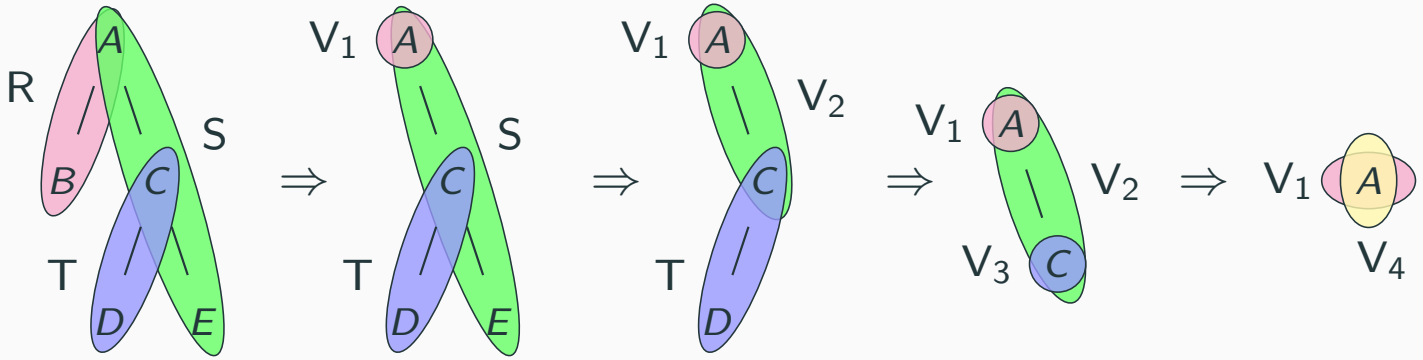

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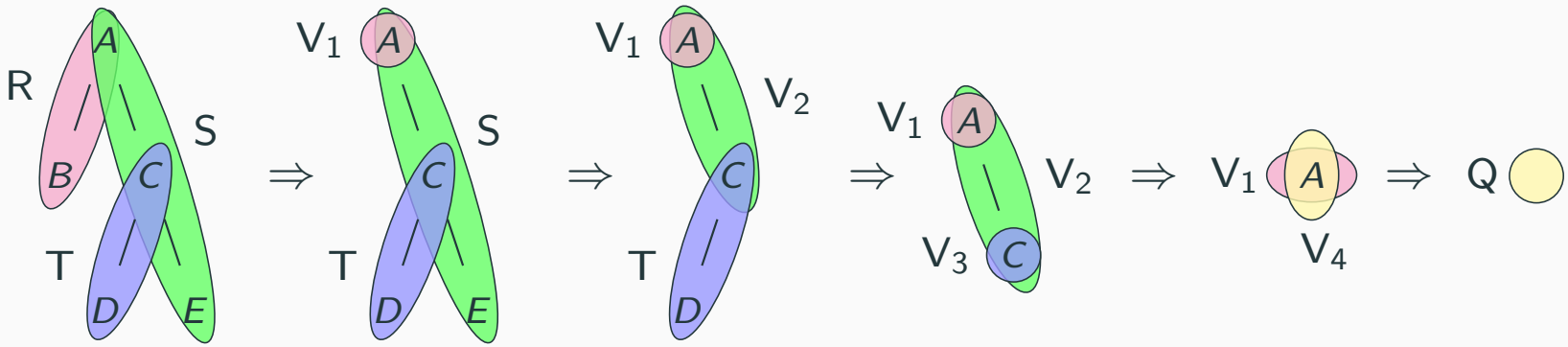

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$$V_4(a) = \sum_c (V_2(a, c) \cdot V_3(c)) \qquad \mathcal{O}(N)$$

$$Q = \sum_a (V_1(a) \cdot V_4(a)) \qquad \mathcal{O}(N)$$

Factor Payloads Enable Different Aggregates (1/2)

Task: Compute $SUM(C \cdot D)$ over the join of R, S, and T

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- Use the same query:

$$\mathrm{Q} = \sum_{a,b,c,d,e} \mathrm{R(a,b)} \cdot \mathrm{S(a,c,e)} \cdot \mathrm{T(c,d)}$$

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Task: Compute $SUM(C \cdot D)$ over the join of R, S, and T

- Use the same query:

$$Q = \sum_{a,b,c,d,e} R(a,b) \cdot S(a,c,e) \cdot T(c,d)$$

- Use factors with different payloads, e.g.,:
  - $R(a,b) = 1$ if $(a,b) \in R$ and 0 otherwise
  - $S(a,c,e) = c$ if $(a,c,e) \in S$ and 0 otherwise
  - $T(c,d) = d$ if $(c,d) \in T$ and 0 otherwise

- For every tuple (a,b,c,d,e),
  we sum up the products $R(a,b) \cdot S(a,c,e) \cdot T(c,d)$

Factor Payloads Enable Different Aggregates (2/2)

- Factors R, S, and T with payloads from a ring $(\mathbf{D}, +, *, \mathbf{0}, \mathbf{1})$
  - Each tuple has the payload $\mathbf{1} \in \mathbf{D}$
- Lift factors Λ_A, Λ_B, Λ_C, Λ_D, Λ_E map the domain of a variable to $\mathbf{D}$

  COUNT all lift factors map to $1 \in \mathbb{Z}$

  SUM(C*D) $\Lambda_C[c] = c \in \mathbb{R}$ and $\Lambda_D[d] = d \in \mathbb{R}$; all others map to $1 \in \mathbb{R}$

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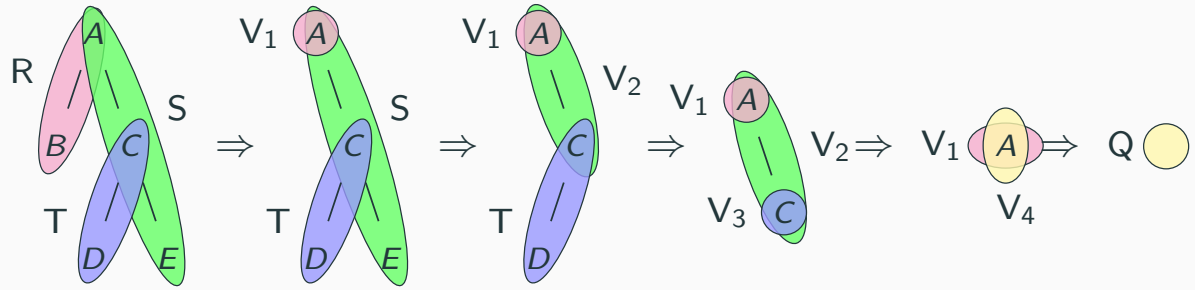
- Lift values of a variable just before its marginalization



$V_1(a) = \sum_b (R(a,b) \cdot \Lambda_B(b))$

$V_2(a,c) = \sum_e (S(a,c,e) \cdot \Lambda_E(e))$

$V_3(c) = \sum_d (T(c,d) \cdot \Lambda_D(d))$

$V_4(a) = \sum_c (V_2(a,c) \cdot V_3(c) \cdot \Lambda_C(c))$

$Q = \sum_a (V_1(a) \cdot V_4(a) \cdot \Lambda_A(a))$

Update Example

- Database of three factors R, S, T

R		
A	B	#
a_1	b_1	2
a_2	b_1	3

S		
B	C	#
b_1	c_1	2
b_1	c_2	1

T		
C	A	#
c_1	a_1	1
c_2	a_1	3
c_2	a_2	3

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c_1	a_1	1
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$R \cdot S \cdot T$			
A	B	C	#
a_1	b_1	c_1	$2 \cdot 2 \cdot 1 = 4$

Update Example

- Database of three factors R, S, T

R

A	B	#
a_1	b_1	2
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T

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c_1	a_1	1
c_2	a_1	3
c_2	a_2	3

$R \cdot S \cdot T$

A	B	C	#
a_1	b_1	c_1	$2 \cdot 2 \cdot 1 = 4$
a_1	b_1	c_2	$2 \cdot 1 \cdot 3 = 6$
a_2	b_1	c_2	$3 \cdot 1 \cdot 3 = 9$

Update Example

- Database of three factors R, S, T
- Triangle Count Query: $Q = \sum_{a,b,c} R(a,b) \cdot S(b,c) \cdot T(c,a)$

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A	B	$\#$
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B	C	$\#$
b_1	c_1	2
b_1	c_2	1

T		
C	A	$\#$
c_1	a_1	1
c_2	a_1	3
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Q	
$\emptyset$	$\#$
$()$	$4 + 6 + 9 = 19$

Update Example

- Database of three factors R, S, T
- Triangle Count Query: $Q = \sum_{a,b,c} R(a,b) \cdot S(b,c) \cdot T(c,a)$
- A single-tuple update is a factor mapping a tuple to a non-zero value (positive for insertions, negative for deletions)

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a_1	b_1	2
a_2	b_1	3

S

B	C	#
b_1	c_1	2
b_1	c_2	1

T

C	A	#
c_1	a_1	1
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↑

$\delta R = \{(a_2, b_1) \mapsto -2\}$

A	B	#
a_2	b_1	-2

↓

Q

$\emptyset$	#
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↑ ↓

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↑

↓

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A	B	#
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Q

$\emptyset$	#
( )	~~$4 + 6 + 9 = 19$~~
( )	$4 + 6 + 3 = 13$

Part 2.

Main IVM techniques:

First-Order IVM

Incremental Computation for Queries

- Input: Query Q, database D, updates δD to D
- Task: Maintain the query result $Q(D)$ under changes δD

$$Q(D + \delta D) = Q(D) + \delta Q(D, \delta D)$$

IVM is faster than re-computation when:

- The "merge" operation of the two query results is fast

  Upserts into hashmaps, appends to lists

- Delta query δQ is faster

  Lower computational complexity, less data input and output

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Question: How can we express the delta queries?

Query Language Closed under Taking Deltas

Given:

- Query Q
- Update $\delta \mathsf{T}$ for factor T in Q

The delta query is defined on the structure of Q:

$$\delta(\mathsf{R} + \mathsf{S}) = \delta\mathsf{R} + \delta\mathsf{S}$$

$$\delta(\mathsf{R} \cdot \mathsf{S}) = (\delta\mathsf{R} \cdot \mathsf{S}) + (\mathsf{R} \cdot \delta\mathsf{S}) + (\delta\mathsf{R} \cdot \delta\mathsf{S})$$

$$\delta(\sum_{a} \mathsf{R}) = \sum_{a} \delta\mathsf{R}$$

$$\delta\mathsf{R} = \begin{cases} \delta\mathsf{T} & \text{if } \mathsf{R} = \mathsf{T} \\ 0 & \text{otherwise } // \text{ Factor 0 maps empty tuple to value 0} \end{cases}$$

Example: Delta for Simple Join Query

Given: Factors R and S of size N

- Query:

$$Q(a, b) = R(a, b) \cdot S(b)$$

Evaluation time: $\mathcal{O}(N)$

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Single-tuple update: $\mathcal{O}(1)$ for $\delta R(a, b)$ and $\mathcal{O}(N)$ for $\delta S(b)$
Enumeration delay: $\mathcal{O}(1)$

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Single-tuple update: $\mathcal{O}(1)$ for $\delta R(a, b)$ and $\mathcal{O}(N)$ for $\delta S(b)$
Enumeration delay: $\mathcal{O}(1)$

Optimal: $\mathcal{O}(1)$ single-tuple update time and enumeration delay

Example: Delta for Simple Project-Join Query

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- Query:

$$Q(a) = \sum_{b} R(a, b) \cdot S(b)$$

Evaluation time: $\mathcal{O}(N)$

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  Evaluation time: $\mathcal{O}(N)$

- Delta queries:

$$\delta Q(a) = \sum_b \delta R(a,b) \cdot S(b)$$

$$\delta Q(a) = \sum_b R(a,b) \cdot \delta S(b)$$

  Single-tuple update time: $\mathcal{O}(1)$ for δR and $\mathcal{O}(N)$ for δS

  Enumeration delay: $\mathcal{O}(1)$

Example: Delta for Simple Project-Join Query

Given: Factors R and S of size N

- Query:

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- Delta queries:

$$\delta Q(a) = \sum_b \delta R(a,b) \cdot S(b)$$

$$\delta Q(a) = \sum_b R(a,b) \cdot \delta S(b)$$

  Single-tuple update time: $\mathcal{O}(1)$ for δR and $\mathcal{O}(N)$ for δS
  Enumeration delay: $\mathcal{O}(1)$

  Optimal: $\mathcal{O}(\sqrt{N})$ single-tuple update time and enumeration delay

Example: Delta for Triangle Join Query (1/3)

Given: Graph with edge factor E, update δE

- Query:

$$Q(a, b, c) = E(a, b) \cdot E(b, c) \cdot E(c, a)$$

Evaluation time: $\mathcal{O}(|E|^{1.5})$ (clarified later in the course)

Example: Delta for Triangle Join Query (1/3)

Given: Graph with edge factor E, update δE

- Query:

$$Q(a,b,c) = E(a,b) \cdot E(b,c) \cdot E(c,a)$$

Evaluation time: $\mathcal{O}(|E|^{1.5})$ (clarified later in the course)

- Delta query:

$$\begin{aligned}\delta Q(a,b,c) &= \delta\left(E(a,b) \cdot E(b,c) \cdot E(c,a)\right) \\ &= \delta E(a,b) \cdot E(b,c) \cdot E(c,a) + E(a,b) \cdot \delta\left(E(b,c) \cdot E(c,a)\right) \\ &+ \delta E(a,b) \cdot \delta\left(E(b,c) \cdot E(c,a)\right)\end{aligned}$$

We next expand:

$$\begin{aligned}\delta\left(E(b,c) \cdot E(c,a)\right) = \delta E(b,c) \cdot E(c,a) + E(b,c) \cdot \delta E(c,a) \\ + \delta E(b,c) \cdot \delta E(c,a)\end{aligned}$$

Example: Delta for Triangle Join Query (2/3)

$$\begin{aligned}\delta Q(a,b,c) = {} & \delta E(a,b) \cdot E(b,c) \cdot E(c,a) + E(a,b) \cdot \delta E(b,c) \cdot E(c,a) \\ & + E(a,b) \cdot E(b,c) \cdot \delta E(c,a) + E(a,b) \cdot \delta E(b,c) \cdot \delta E(c,a) \\ & + \delta E(a,b) \cdot \delta E(b,c) \cdot E(c,a) + \delta E(a,b) \cdot E(b,c) \cdot \delta E(c,a) \\ & + \delta E(a,b) \cdot \delta E(b,c) \cdot \delta E(c,a)\end{aligned}$$

Example: Delta for Triangle Join Query (2/3)

$$\begin{aligned}\delta Q(a,b,c) = {} & \delta E(a,b) \cdot E(b,c) \cdot E(c,a) + E(a,b) \cdot \delta E(b,c) \cdot E(c,a) \\ & + E(a,b) \cdot E(b,c) \cdot \delta E(c,a) + E(a,b) \cdot \delta E(b,c) \cdot \delta E(c,a) \\ & + \delta E(a,b) \cdot \delta E(b,c) \cdot E(c,a) + \delta E(a,b) \cdot E(b,c) \cdot \delta E(c,a) \\ & + \delta E(a,b) \cdot \delta E(b,c) \cdot \delta E(c,a)\end{aligned}$$

Can we simplify this query?

Example: Delta for Triangle Join Query (2/3)

$$\begin{aligned}\delta Q(a,b,c) = {} & \delta E(a,b) \cdot E(b,c) \cdot E(c,a) + E(a,b) \cdot \delta E(b,c) \cdot E(c,a) \\ & + E(a,b) \cdot E(b,c) \cdot \delta E(c,a) + E(a,b) \cdot \delta E(b,c) \cdot \delta E(c,a) \\ & + \delta E(a,b) \cdot \delta E(b,c) \cdot E(c,a) + \delta E(a,b) \cdot E(b,c) \cdot \delta E(c,a) \\ & + \delta E(a,b) \cdot \delta E(b,c) \cdot \delta E(c,a)\end{aligned}$$

Can we simplify this query?

Yes! Idea: Q is symmetric, so

$$\begin{aligned}\delta E(a,b) \cdot E(b,c) \cdot E(c,a) &= E(a,b) \cdot \delta E(b,c) \cdot E(c,a) \\ &= E(a,b) \cdot E(b,c) \cdot \delta E(c,a)\end{aligned}$$

Example: Delta for Triangle Join Query (3/3)

$$\begin{aligned}\delta Q(a,b,c) = {} & \delta E(a,b) \cdot E(b,c) \cdot E(c,a) + E(a,b) \cdot \delta E(b,c) \cdot E(c,a) \\ & + E(a,b) \cdot E(b,c) \cdot \delta E(c,a) + E(a,b) \cdot \delta E(b,c) \cdot \delta E(c,a) \\ & + \delta E(a,b) \cdot \delta E(b,c) \cdot E(c,a) + \delta E(a,b) \cdot E(b,c) \cdot \delta E(c,a) \\ & + \delta E(a,b) \cdot \delta E(b,c) \cdot \delta E(c,a)\end{aligned}$$

Let 3 be a factor mapping the empty tuple to multiplicity 3. Then,

$$\begin{aligned}\delta Q(a,b,c) = {} & 3 \cdot \delta E(a,b) \cdot E(b,c) \cdot E(c,a) \\ & + 3 \cdot E(a,b) \cdot \delta E(b,c) \cdot \delta E(c,a) \\ & + \delta E(a,b) \cdot \delta E(b,c) \cdot \delta E(c,a)\end{aligned}$$

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Single-tuple update time: $\mathcal{O}(|E|)$ Enumeration delay: $\mathcal{O}(1)$

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Single-tuple update time: $\mathcal{O}(|E|)$ Enumeration delay: $\mathcal{O}(1)$

Optimal: $\mathcal{O}(\sqrt{|E|})$ single-tuple update time and $\mathcal{O}(1)$ delay

Example: Delta for Triangle Count Query

Graph with edge factor E, update δE

- Query:

$$Q = \sum_{a,b,c} E(a,b) \cdot E(b,c) \cdot E(c,a)$$

Evaluation time: $\mathcal{O}(|E|^{1.5})$. Strassen-like algorithm: $\mathcal{O}(|E|^{1.41})$

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Given: Factors R, S, T of size N

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  Evaluation time: $\mathcal{O}(N^2)$

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$$\begin{aligned}
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&= \sum_{d,e} \delta \left(R(a,b) \cdot S(a,c,e) \cdot T(a,c,d) \right) \\
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\end{aligned}$$

Single-tuple update time: $\mathcal{O}(N)$. Enumeration delay: $\mathcal{O}(1)$

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\end{aligned}$$

  Single-tuple update time: $\mathcal{O}(N)$. Enumeration delay: $\mathcal{O}(1)$

  Optimal: $\mathcal{O}(1)$ single-tuple update time and delay

Beyond Delta Queries

Can we achieve
the aforementioned optimal update times
by using additional storage?

Part 2.

Main IVM techniques:

Higher-Order IVM

Using Materialized Views To Lower Update Time

We reconsider the following query with factors R and S of size N

- Query:

$$Q(a, b) = R(a, b) \cdot S(b)$$

Using Materialized Views To Lower Update Time

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$$\delta Q(a, b_0) = R(a, b_0) \cdot \delta S(b_0)$$

Single-tuple update: $\mathcal{O}(N)$, Enumeration delay: $\mathcal{O}(1)$

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Single-tuple update: $\mathcal{O}(N)$, Enumeration delay: $\mathcal{O}(1)$

Using the views

$$V_R(b) = \sum_a R(a, b) \quad \text{and} \quad V_{RS}(b) = V_R(b) \cdot S(b)$$

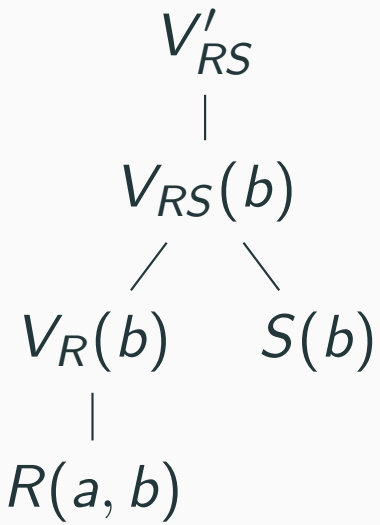
we achieve optimality:

Single-tuple update: $\mathcal{O}(1)$, Enumeration delay: $\mathcal{O}(1)$

Tree of Materialized Views

We organize the input factors and the extra views in a tree

View tree



Views

$$V'_{RS} = \sum_b V_{RS}(b)$$

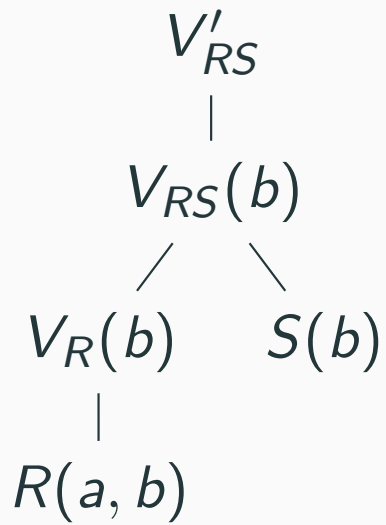
$$V_{RS}(b) = V_R(b) \cdot S(b)$$

$$V_R(b) = \sum_a R(a,b)$$

Computation time: $\mathcal{O}(N)$

Maintaining View Tree Under Updates (1/2)

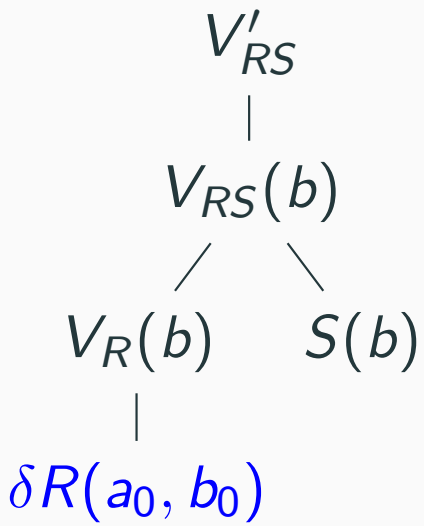
View tree



Single-tuple update $\delta R(a_0, b_0)$

Maintaining View Tree Under Updates (1/2)

View tree



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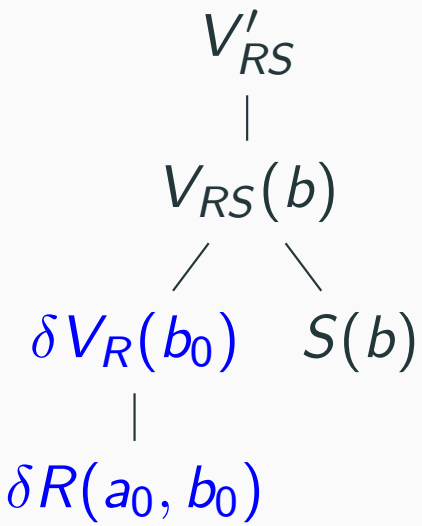

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$$\delta V_R(b_0) = \sum_{a_0} \delta R(a_0, b_0) = \delta R(a_0, b_0)$$

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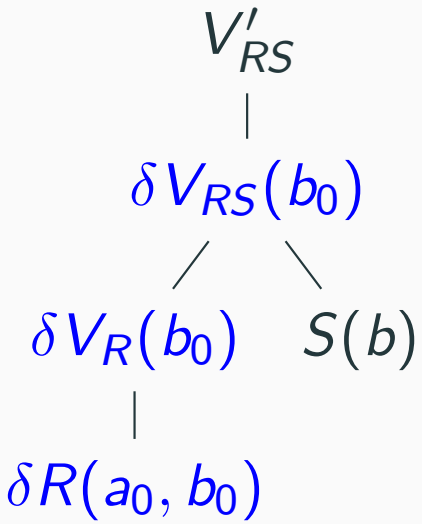

Single-tuple update $\delta R(a_0, b_0)$

$$\delta V_{RS}(b_0) = \delta V_R(b_0) \cdot S(b_0)$$

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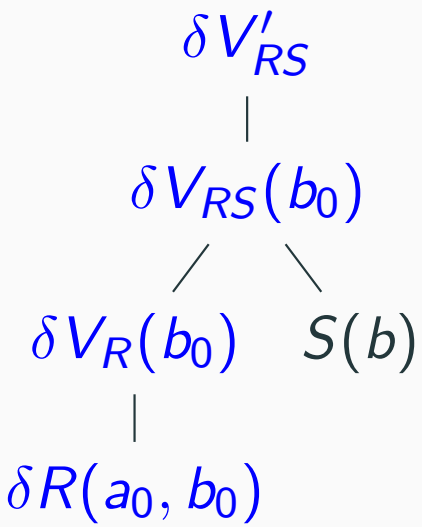

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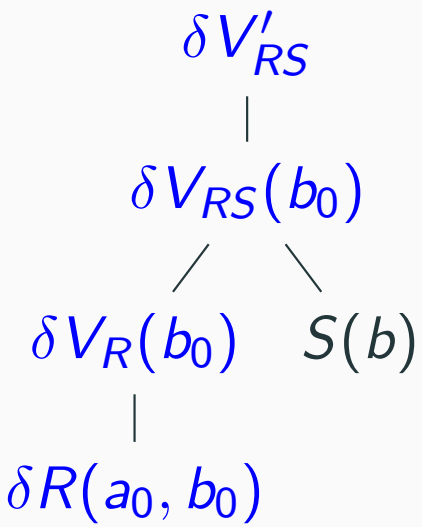

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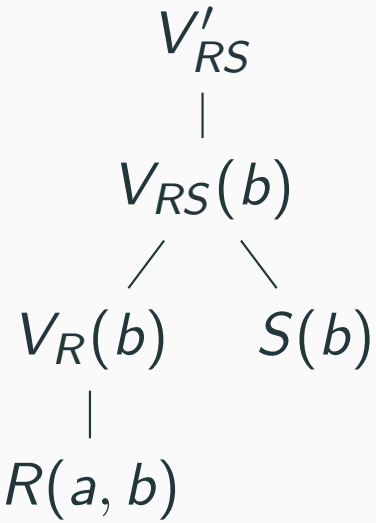
Update time: O(1)

Maintaining View Tree Under Updates (2/2)

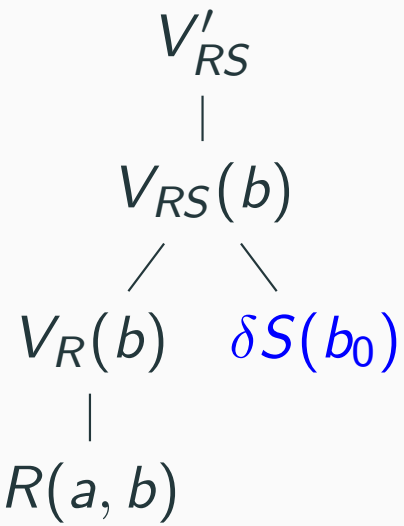
View tree



Single-tuple update $\delta S(b_0)$

Maintaining View Tree Under Updates (2/2)

View tree



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Maintaining View Tree Under Updates (2/2)

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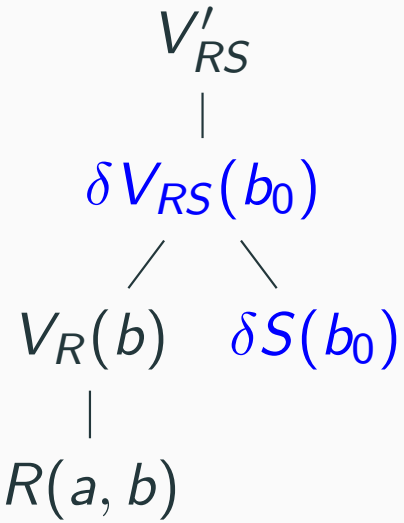

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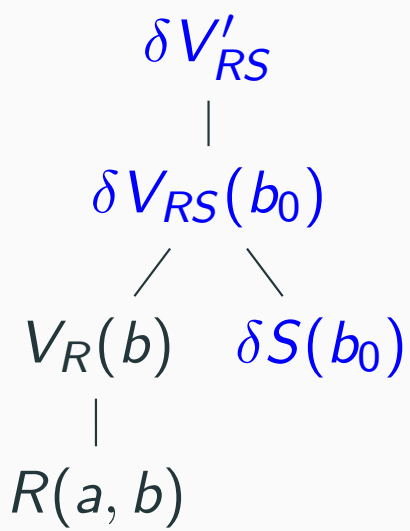

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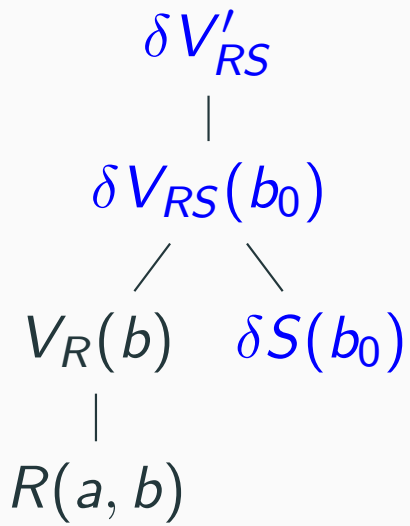

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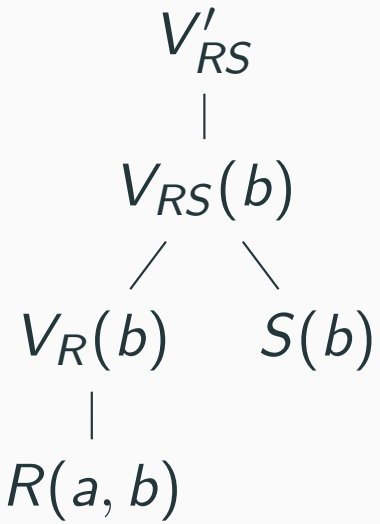
Enumeration from View Tree

Enumeration

View tree



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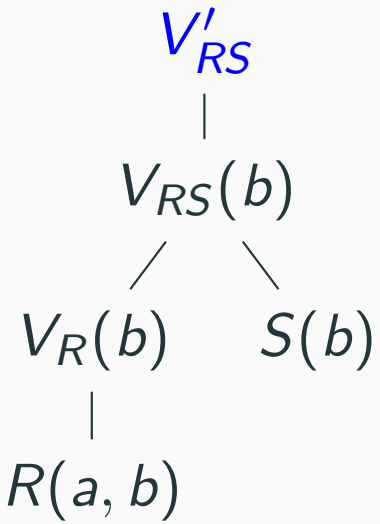

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- Is V'_{RS} empty? If yes, stop!

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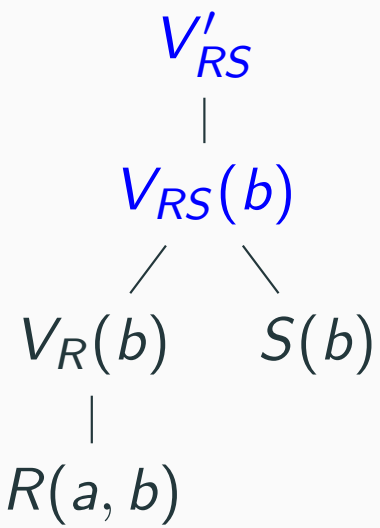

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- Is V'_{RS} empty? If yes, stop!
- Iterate over (distinct) b's in V_{RS}

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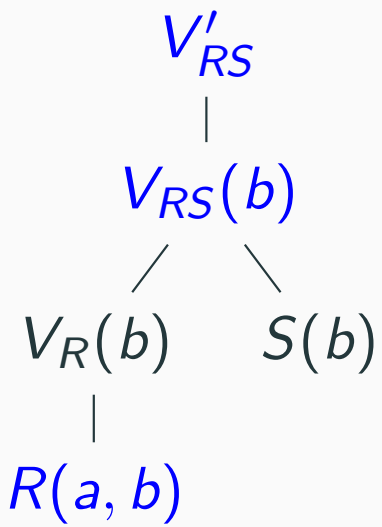

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Enumeration from View Tree

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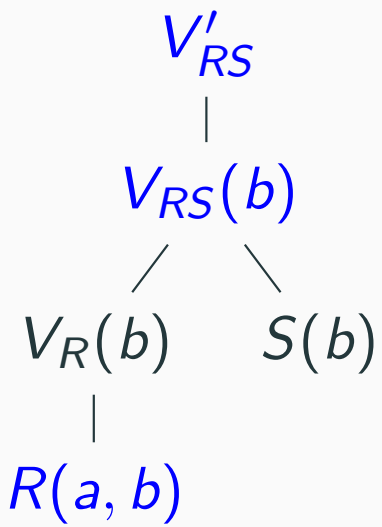

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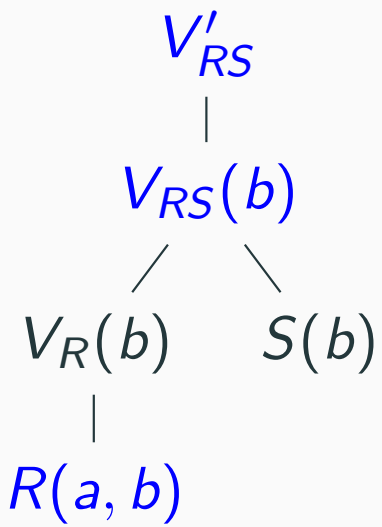

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Enumeration delay: $\mathcal{O}(1)$

How to Generalize This Example?

We need to construct view trees for the query

- Similar to query plans for static query evaluation
- Goal: Minimize the propagation time for each update

We next sketch two main approaches

- DBToaster `https://dbtoaster.github.io`
- F-IVM `https://github.com/fdbresearch/FIVM`

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  1. One view tree per updatable factor, which is child of root
  2. Materialized view over each connected component of the remaining factors
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  4. This view is treated like a query (Go to 1)
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- F-IVM `https://github.com/fdbresearch/FIVM`
  1. One view tree for all updatable factors
  2. Structure of view tree follows a partial order of query variables
  3. Query result possibly distributed over several views (previous example)

Extra Slides:

DBToaster Example

Higher-Order IVM using DBToaster (1/10)

Consider again the factors R, S, T of size N and the query

$$Q(a, b, c) = \sum_{d,e} R(a, b) \cdot S(a, c, e) \cdot T(a, c, d)$$

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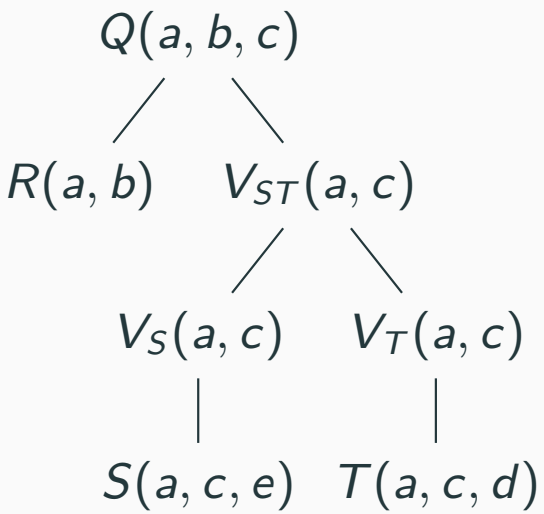
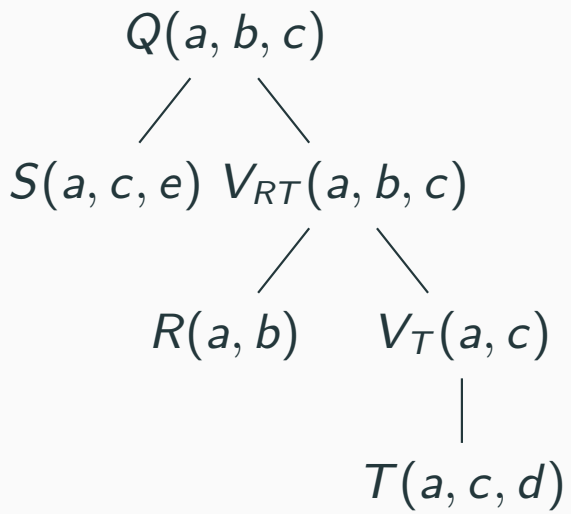
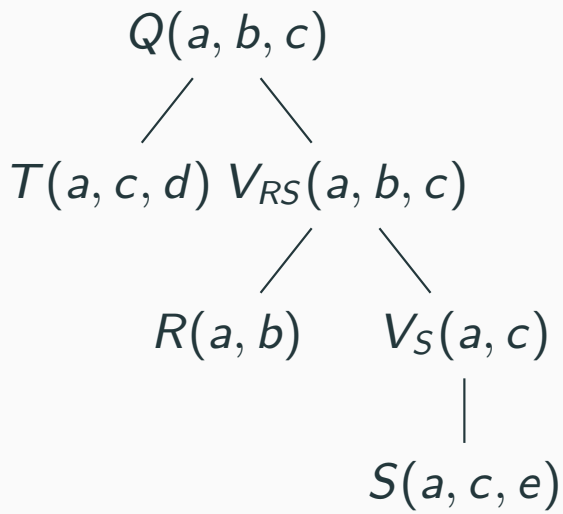
DBToaster constructs a view tree for each factor:



For update δR to R



For update δS to S



For update δT to T

Higher-Order IVM using DBToaster (2/10)

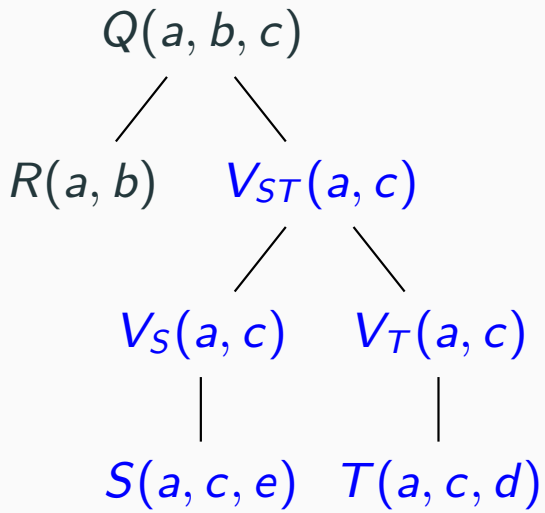
$$Q(a,b,c) = \sum_{d,e} R(a,b) \cdot S(a,c,e) \cdot T(a,c,d)$$

For update δR to R:

DBToaster creates the materialized view V_{ST}

$$\begin{aligned} V_{ST}(a,c) &= \sum_{d,e} S(a,c,e) \cdot T(a,c,d) \\ &= \sum_{e} S(a,c,e) \cdot \sum_{d} T(a,c,d) \\ &= V_S(a,c) \cdot V_T(a,c) \end{aligned}$$

Computation time: $\mathcal{O}(N)$



Higher-Order IVM using DBToaster (2/10)

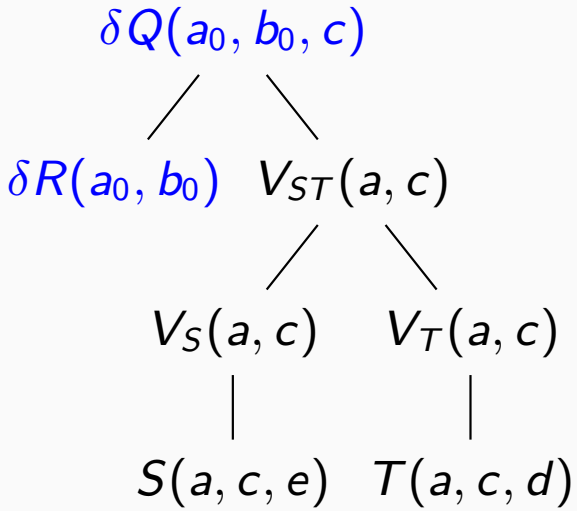
$$Q(a,b,c) = \sum_{d,e} R(a,b) \cdot S(a,c,e) \cdot T(a,c,d)$$

For update δR to R:

DBToaster creates the materialized view V_{ST}

$$\begin{aligned} V_{ST}(a,c) &= \sum_{d,e} S(a,c,e) \cdot T(a,c,d) \\ &= \sum_{e} S(a,c,e) \cdot \sum_{d} T(a,c,d) \\ &= V_S(a,c) \cdot V_T(a,c) \end{aligned}$$

Computation time: $\mathcal{O}(N)$



The delta query for update $\delta R(a_0, b_0)$ using V_{ST} is then:

$$\delta Q(a,b,c) = \delta R(a_0,b_0) \cdot V_{ST}(a_0,c)$$

Single-tuple update time: $\mathcal{O}(N)$

Higher-Order IVM using DBToaster (3/10)

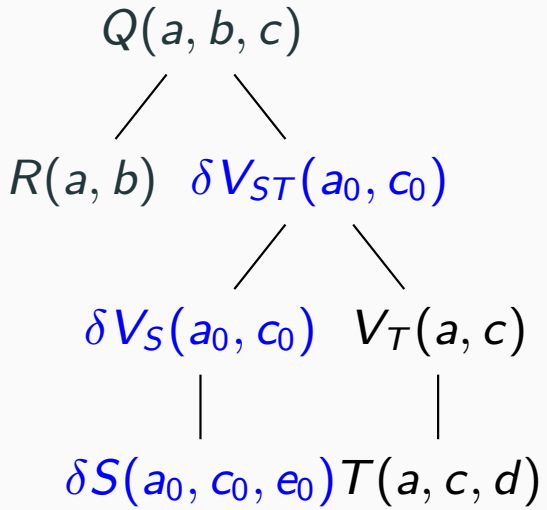
$$Q(a,b,c) = \sum_{d,e} R(a,b) \cdot S(a,c,e) \cdot T(a,c,d)$$

The materialized view V_{ST} needs to be updated, too

Delta query for update $\delta S(a_0, c_0, e_0)$:

$$\delta V_S(a_0, c_0) = \sum_{e_0} \delta S(a_0, c_0, e_0) = \delta S(a_0, c_0, e_0)$$

$$\delta V_{ST}(a_0, c_0) = \delta V_S(a_0, c_0) \cdot V_T(a_0, c_0)$$



Higher-Order IVM using DBToaster (3/10)

$$Q(a,b,c) = \sum_{d,e} R(a,b) \cdot S(a,c,e) \cdot T(a,c,d)$$

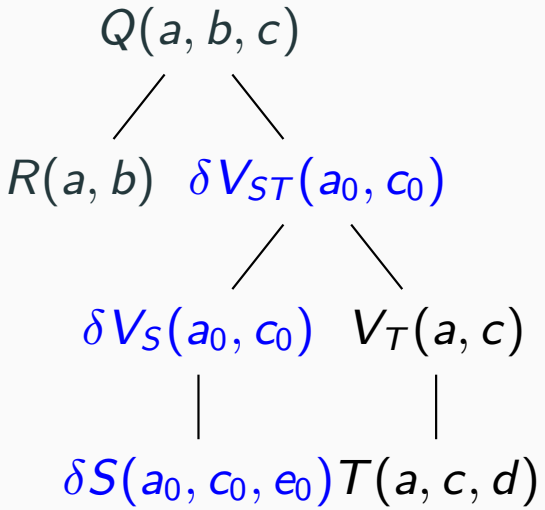
The materialized view V_{ST} needs to be updated, too

Delta query for update $\delta S(a_0, c_0, e_0)$:

$$\delta V_S(a_0, c_0) = \sum_{e_0} \delta S(a_0, c_0, e_0) = \delta S(a_0, c_0, e_0)$$

$$\delta V_{ST}(a_0, c_0) = \delta V_S(a_0, c_0) \cdot V_T(a_0, c_0)$$

Single-tuple update time: $\mathcal{O}(1)$



Higher-Order IVM using DBToaster (4/10)

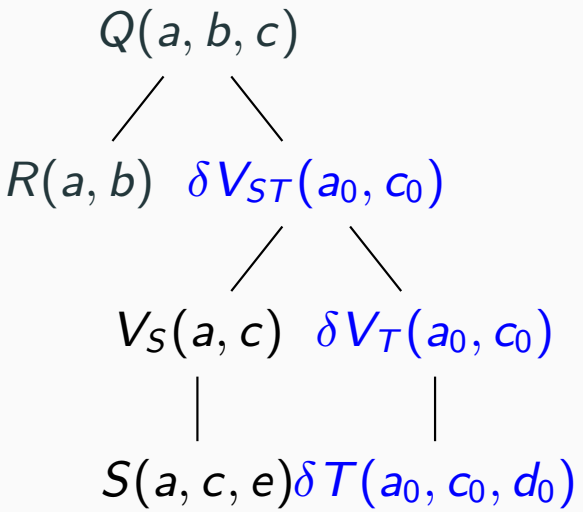
$$Q(a,b,c) = \sum_{d,e} R(a,b) \cdot S(a,c,e) \cdot T(a,c,d)$$

The materialized view V_{ST} needs to be updated, too

Delta query for update $\delta T(a_0, c_0, d_0)$:

$$\delta V_T(a_0, c_0) = \sum_{d_0} \delta T(a_0, c_0, d_0) = \delta T(a_0, c_0, d_0)$$

$$\delta V_{ST}(a_0, c_0) = V_S(a_0, c_0) \cdot \delta V_T(a_0, c_0)$$



Higher-Order IVM using DBToaster (4/10)

$$Q(a,b,c) = \sum_{d,e} R(a,b) \cdot S(a,c,e) \cdot T(a,c,d)$$

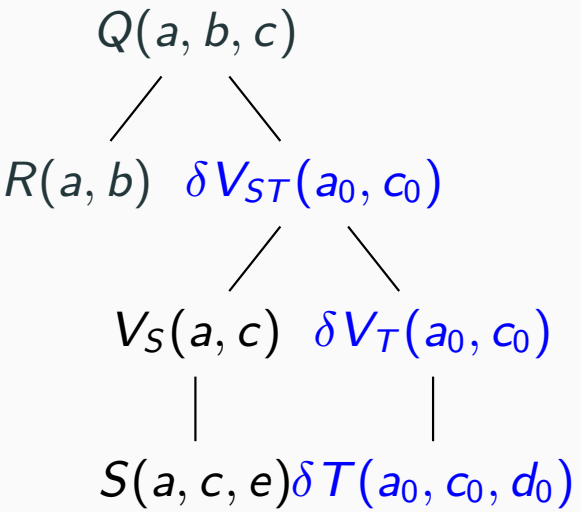
The materialized view V_{ST} needs to be updated, too

Delta query for update $\delta T(a_0, c_0, d_0)$:

$$\delta V_T(a_0, c_0) = \sum_{d_0} \delta T(a_0, c_0, d_0) = \delta T(a_0, c_0, d_0)$$

$$\delta V_{ST}(a_0, c_0) = V_S(a_0, c_0) \cdot \delta V_T(a_0, c_0)$$

Single-tuple update time: $\mathcal{O}(1)$



Higher-Order IVM using DBToaster (5/10)

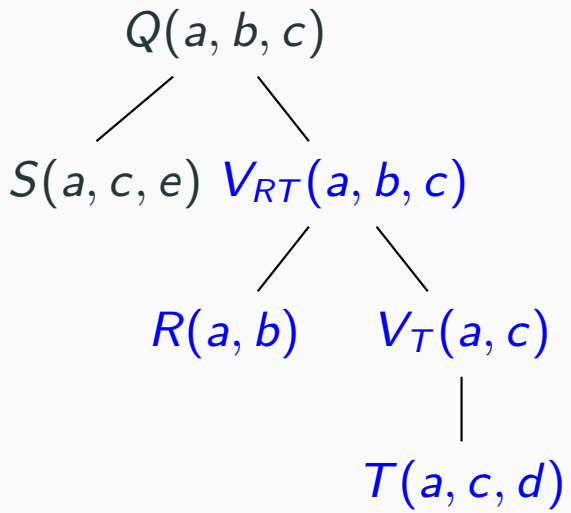
$$Q(a,b,c) = \sum_{d,e} R(a,b) \cdot S(a,c,e) \cdot T(a,c,d)$$

For update δS to S:

DBToaster creates the materialized view V_{RT}

$$\begin{aligned} V_{RT}(a,b,c) &= \sum_{d} R(a,b) \cdot T(a,c,d) \\ &= R(a,b) \cdot \sum_{d} T(a,c,d) \\ &= R(a,b) \cdot V_T(a,c) \end{aligned}$$

Computation time: $\mathcal{O}(N^2)$



Higher-Order IVM using DBToaster (5/10)

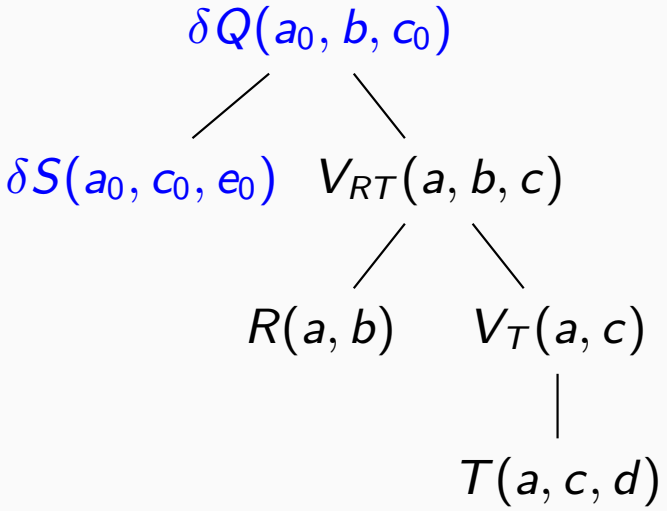
$$Q(a,b,c) = \sum_{d,e} R(a,b) \cdot S(a,c,e) \cdot T(a,c,d)$$

For update δS to S:

DBToaster creates the materialized view V_{RT}

$$\begin{aligned} V_{RT}(a,b,c) &= \sum_{d} R(a,b) \cdot T(a,c,d) \\ &= R(a,b) \cdot \sum_{d} T(a,c,d) \\ &= R(a,b) \cdot V_T(a,c) \end{aligned}$$

Computation time: $\mathcal{O}(N^2)$



The delta query for update $\delta S(a_0, c_0, e_0)$ using V_{RT} is:

$$\begin{aligned} \delta Q(a_0,b,c_0) &= \sum_{e_0} \delta S(a_0,c_0,e_0) \cdot V_{RT}(a_0,b,c_0) \\ &= \delta S(a_0,c_0,e_0) \cdot V_{RT}(a_0,b,c_0) \end{aligned}$$

Single-tuple update time: $\mathcal{O}(N)$

Higher-Order IVM using DBToaster (6/10)

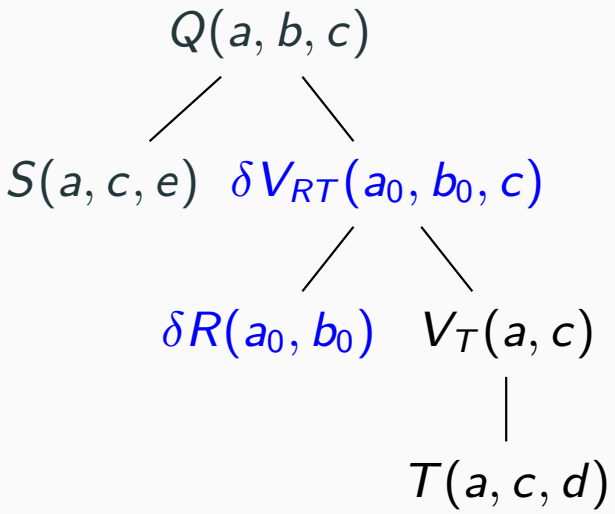
$$Q(a,b,c) = \sum_{d,e} R(a,b) \cdot S(a,c,e) \cdot T(a,c,d)$$

The materialized view V_{RT} needs to be updated, too

Delta query for update $\delta R(a_0, b_0)$:

$$\delta V_{RT}(a_0, b_0, c) = \delta R(a_0, b_0) \cdot V_T(a_0, c)$$

Single-tuple update time: $\mathcal{O}(N)$



Higher-Order IVM using DBToaster (7/10)

$$Q(a,b,c) = \sum_{d,e} R(a,b) \cdot S(a,c,e) \cdot T(a,c,d)$$

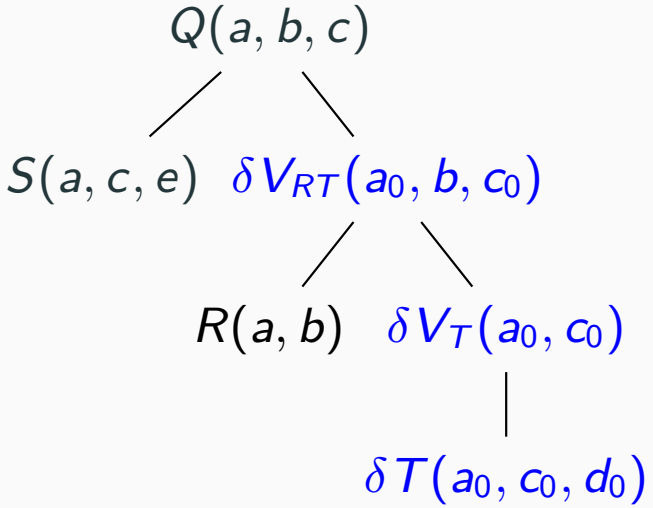
The materialized view V_{RT} needs to be updated, too

Delta queries for update $\delta T(a_0, c_0, d_0)$:

$$\delta V_T(a_0, c_0) = \sum_{d_0} \delta T(a_0, c_0, d_0) = \delta T(a_0, c_0, d_0)$$

$$\delta V_{RT}(a_0, b, c_0) = R(a_0, b) \cdot \delta V_T(a_0, c_0)$$

Single-tuple update time: $\mathcal{O}(1)$ and $\mathcal{O}(N)$, respectively



Higher-Order IVM using DBToaster (8/10)

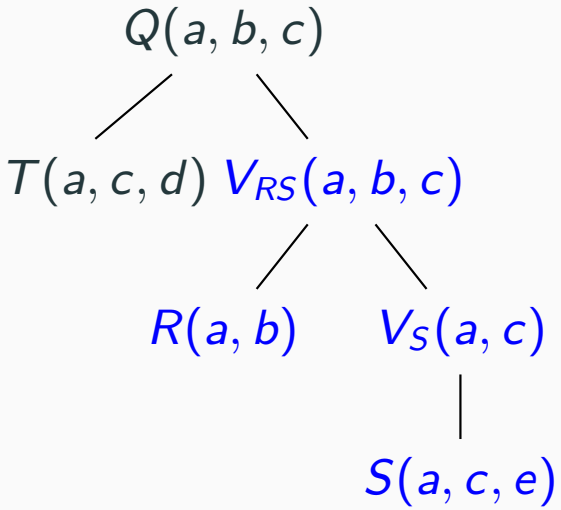
$$Q(a,b,c) = \sum_{d,e} R(a,b) \cdot S(a,c,e) \cdot T(a,c,d)$$

For update δT to T (analogous to update to S):

DBToaster creates the materialized view V_{RS}

$$\begin{aligned} V_{RS}(a,b,c) &= \sum_{e} R(a,b) \cdot S(a,c,e) \\ &= R(a,b) \cdot \sum_{e} S(a,c,e) \\ &= R(a,b) \cdot V_S(a,c) \end{aligned}$$

Computation time: $\mathcal{O}(N^2)$



Higher-Order IVM using DBToaster (8/10)

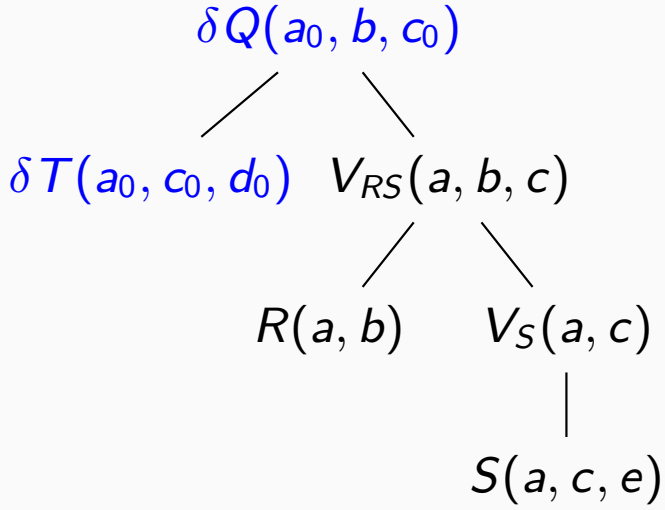
$$Q(a,b,c) = \sum_{d,e} R(a,b) \cdot S(a,c,e) \cdot T(a,c,d)$$

For update δT to T (analogous to update to S):

DBToaster creates the materialized view V_{RS}

$$\begin{aligned} V_{RS}(a,b,c) &= \sum_{e} R(a,b) \cdot S(a,c,e) \\ &= R(a,b) \cdot \sum_{e} S(a,c,e) \\ &= R(a,b) \cdot V_S(a,c) \end{aligned}$$

Computation time: $\mathcal{O}(N^2)$



The delta query for update $\delta T(a_0, c_0, d_0)$ using V_{RS}:

$$\begin{aligned} \delta Q(a_0,b,c_0) &= \sum_{d_0} \delta T(a_0,c_0,d_0) \cdot V_{RS}(a_0,b,c_0) \\ &= \delta T(a_0,c_0,d_0) \cdot V_{RS}(a_0,b,c_0) \end{aligned}$$

Single-tuple update time: $\mathcal{O}(N)$

Higher-Order IVM using DBToaster (9/10)

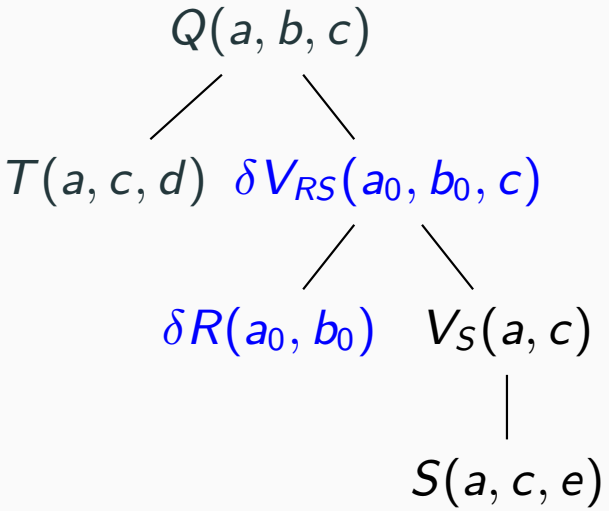
$$Q(a,b,c) = \sum_{d,e} R(a,b) \cdot S(a,c,e) \cdot T(a,c,d)$$

The materialized view V_{RS} needs to be updated, too

Delta query for update $\delta R(a_0, b_0)$:

$$\delta V_{RS}(a_0, b_0, c) = \delta R(a_0, b_0) \cdot V_S(a_0, c)$$

Single-tuple update time: $\mathcal{O}(N)$



Higher-Order IVM using DBToaster (10/10)

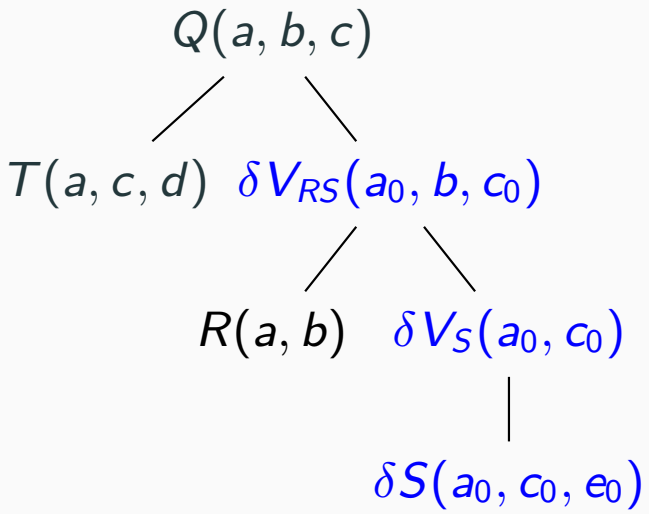
$$Q(a,b,c) = \sum_{d,e} R(a,b) \cdot S(a,c,e) \cdot T(a,c,d)$$

The materialized view V_{RS} needs to be updated, too

Delta queries for update $\delta S(a_0, c_0, e_0)$:

$$\delta V_S(a_0, c_0) = \sum_{e_0} \delta S(a_0, c_0, e_0) = \delta S(a_0, c_0, e_0)$$

$$\delta V_{RS}(a_0, b, c_0) = R(a_0, b) \cdot \delta V_S(a_0, c_0)$$



Single-tuple update time: $\mathcal{O}(1)$ and $\mathcal{O}(N)$, respectively

DBToaster's Update Time Can Be Sub-Optimal

Two sources of sub-optimality (among others)

1. Too many views and view trees to maintain

2. The entire (super-linear) query result available at root views

DBToaster's Update Time Can Be Sub-Optimal

Two sources of sub-optimality (among others)

1. Too many views and view trees to maintain
2. The entire (super-linear) query result available at root views

How to overcome them?

1. One view tree for all updatable factors

   $\Rightarrow$ Share views across updates

2. Keep the query result distributed over several views

   $\Rightarrow$ Factorized representation of the query result

Both features are supported by F-IVM

End of Extra Slides

Higher-Order IVM using F-IVM

F-IVM uses a partial order on the query variables:

- The variables of a factor are on the same path

Challenge: Find variable order that guarantees asymptotically minimal update time (among all variable orders)

- Hard problem already for static query evaluation, more in the FAQ lecture

Higher-Order IVM using F-IVM

F-IVM uses a partial order on the query variables:

- The variables of a factor are on the same path

Challenge: Find variable order that guarantees asymptotically minimal update time (among all variable orders)

- Hard problem already for static query evaluation, more in the FAQ lecture

F-IVM creates a view tree for a given variable order:

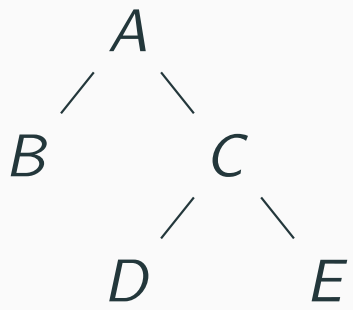
- Each variable induces extra views
- The views join the child views & marginalize the variables
- The input factors are the leaves

Higher-Order IVM using F-IVM: Variable Order

Consider again the factors R, S, T of size N and the query

$$Q(a, b, c) = \sum_{d,e} R(a, b) \cdot S(a, c, e) \cdot T(a, c, d)$$

A possible variable order:



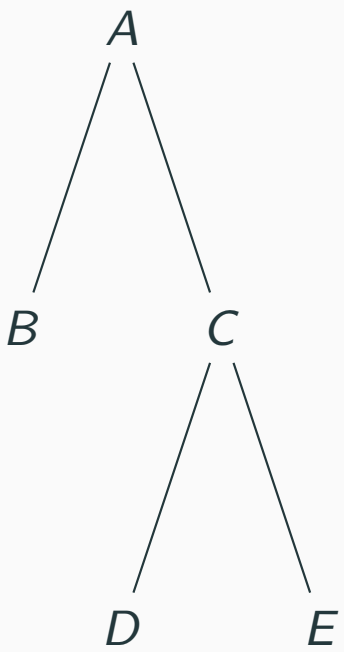
- A is root as it dominates all other variables
  - Dominate: It appears in all factors of the dominated variables
  - B is independent of the other variables given A
- D and E are under C as they are dominated by C
  - D and E are independent given C

Higher-Order IVM using F-IVM: View Tree

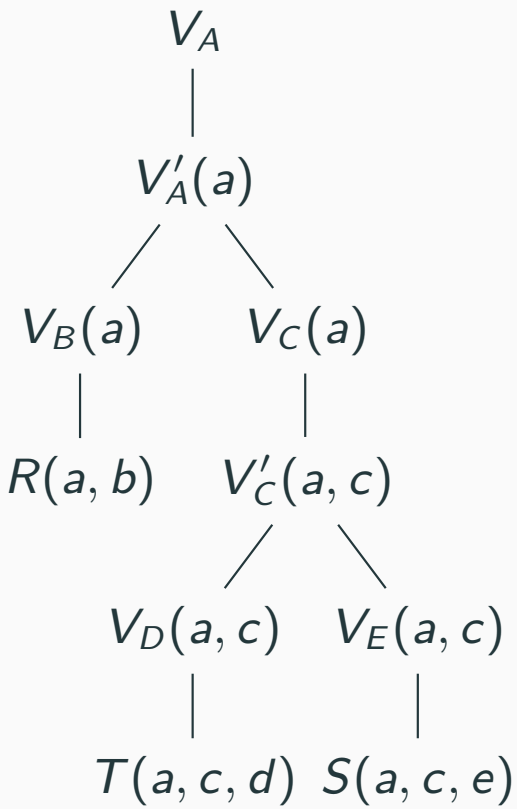
View tree construction:

- Place factors at leaves

Variable order

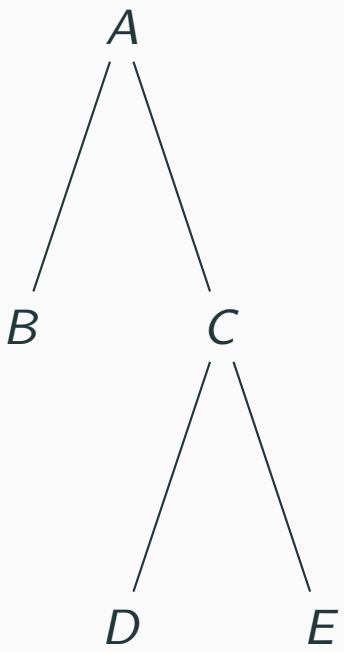


View tree

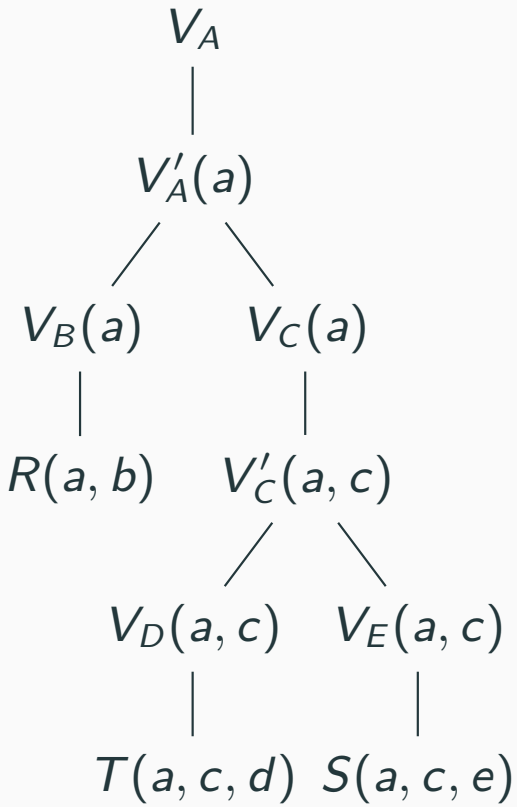


Higher-Order IVM using F-IVM: View Tree

Variable order



View tree



View tree construction:

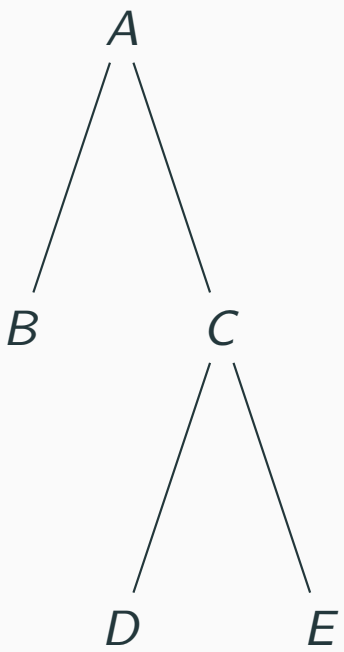
- Place factors at leaves
- Create parent view to join children

$$V'_C(a,c) = V_D(a,c) \cdot V_E(a,c)$$

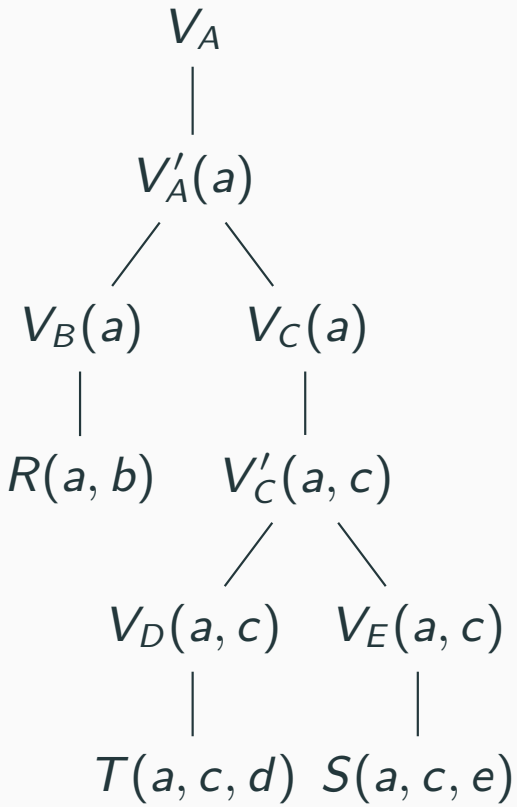
$$V'_A(a) = V_B(a) \cdot V_C(a)$$

Higher-Order IVM using F-IVM: View Tree

Variable order



View tree



View tree construction:

- Place factors at leaves
- Create parent view to join children

$$V'_C(a,c) = V_D(a,c) \cdot V_E(a,c)$$

$$V'_A(a) = V_B(a) \cdot V_C(a)$$

- Aggregate away variables not needed for further joins

$$V_A = \sum_a V'_A(a)$$

$$V_B(a) = \sum_b R(a,b)$$

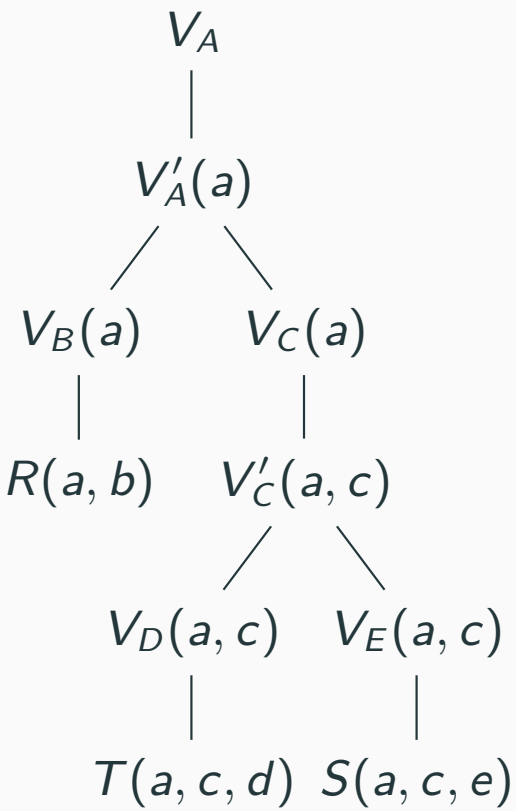
$$V_C(a) = \sum_c V'_C(a,c)$$

$$V_D(a,c) = \sum_d T(a,c,d)$$

$$V_E(a,c) = \sum_e S(a,c,e)$$

Higher-Order IVM using F-IVM: Updates (1/3)

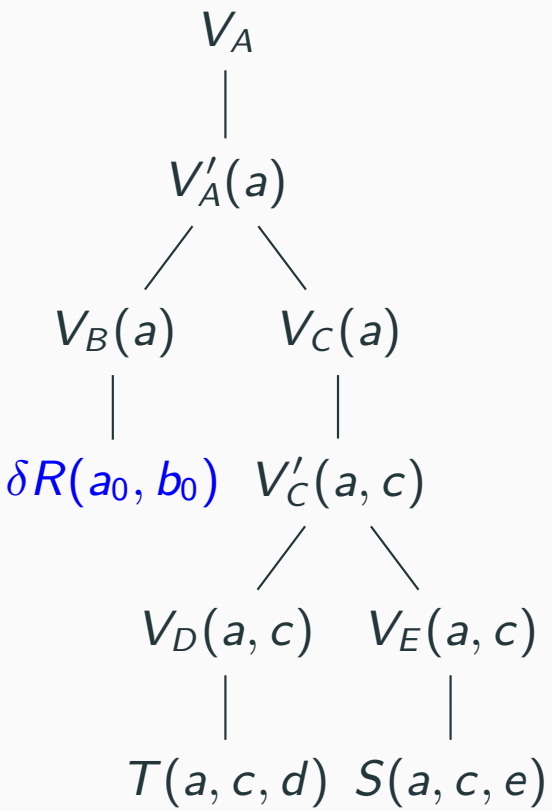
View tree



Single-tuple update $\delta R(a_0, b_0)$ to R:

Higher-Order IVM using F-IVM: Updates (1/3)

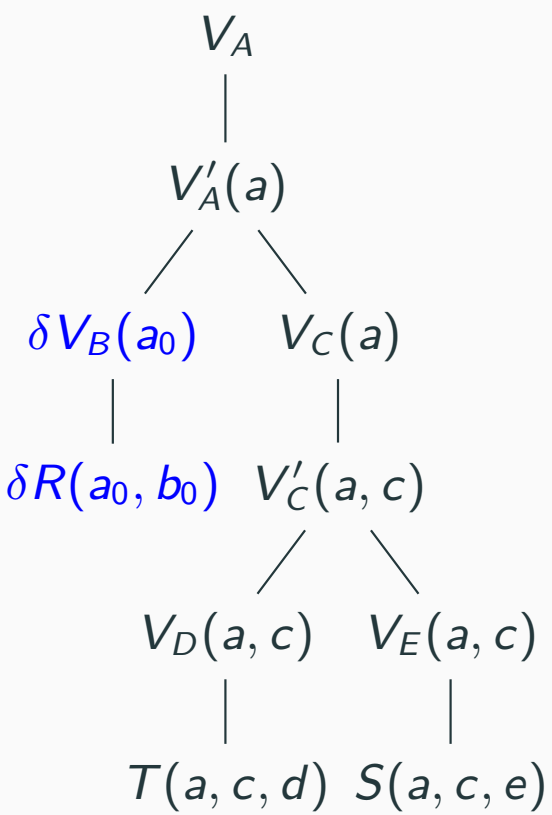
View tree



Single-tuple update $\delta R(a_0, b_0)$ to R:

Higher-Order IVM using F-IVM: Updates (1/3)

View tree



Single-tuple update $\delta R(a_0, b_0)$ to R:

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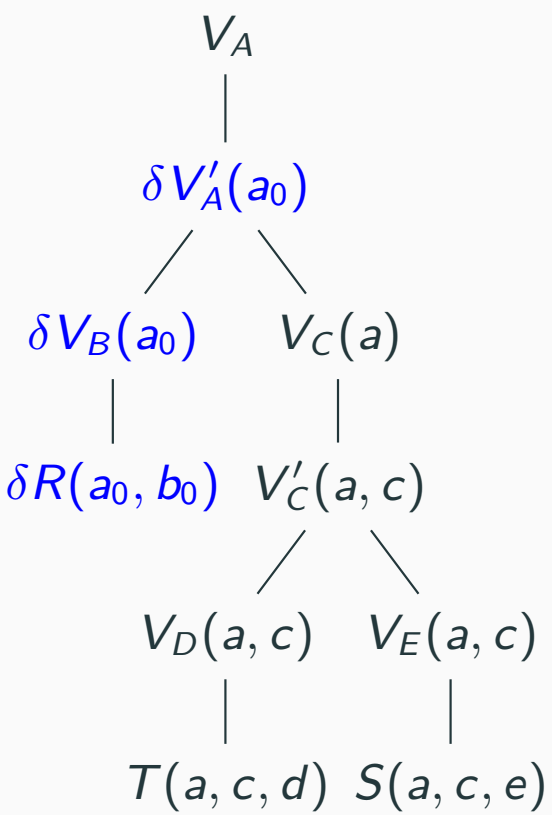

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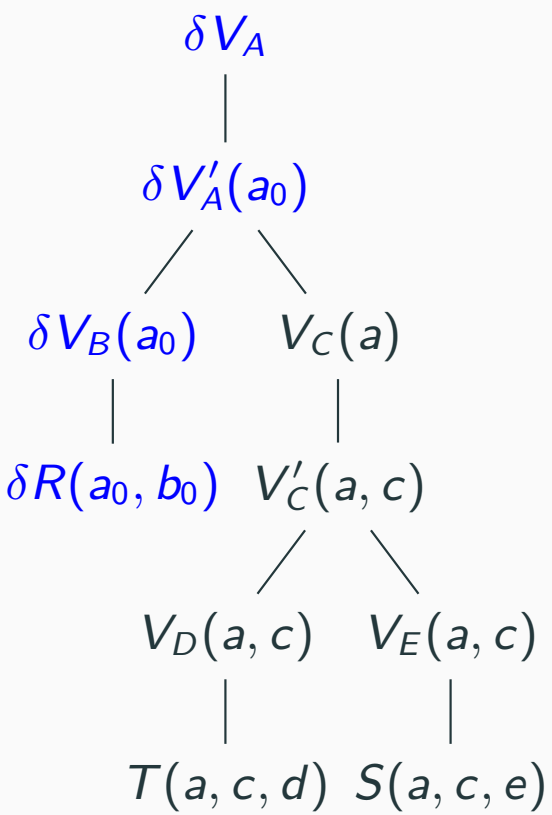

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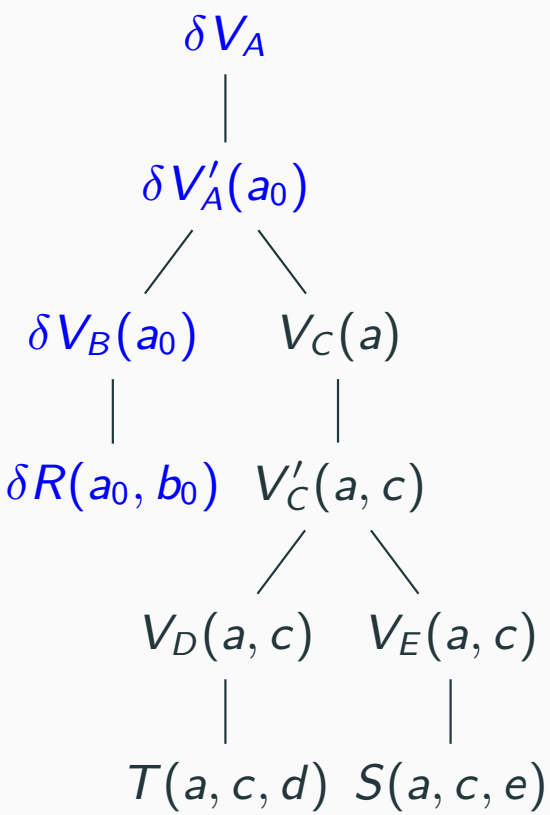

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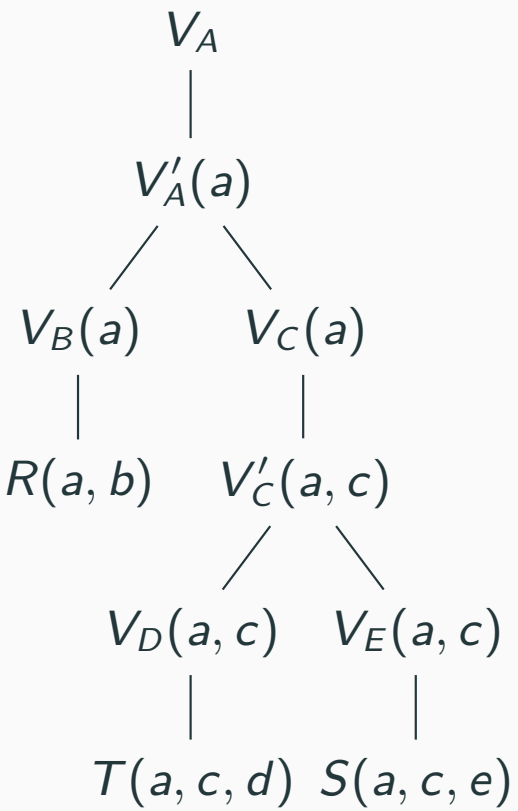

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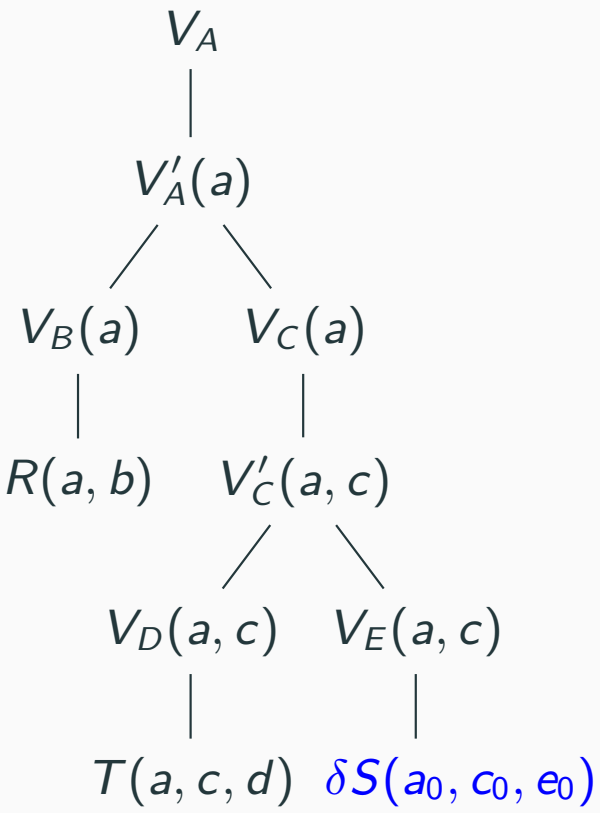

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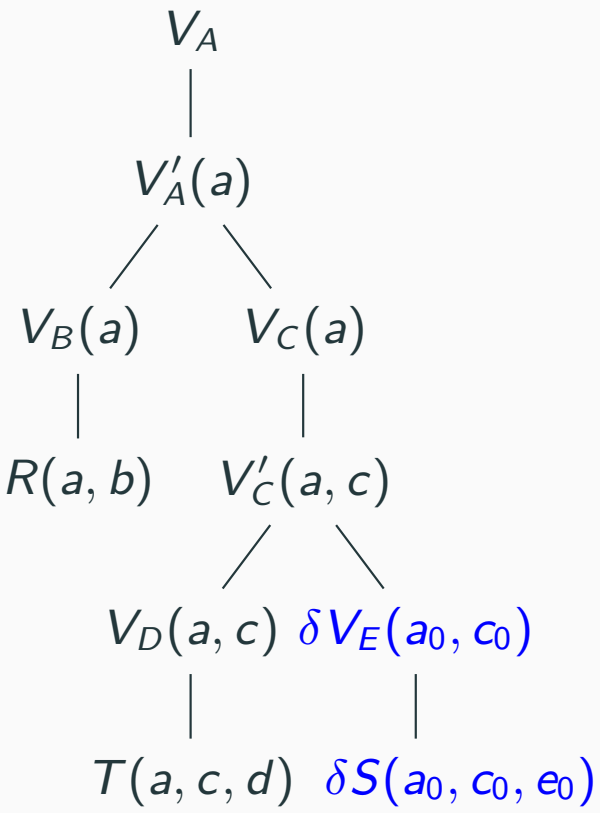

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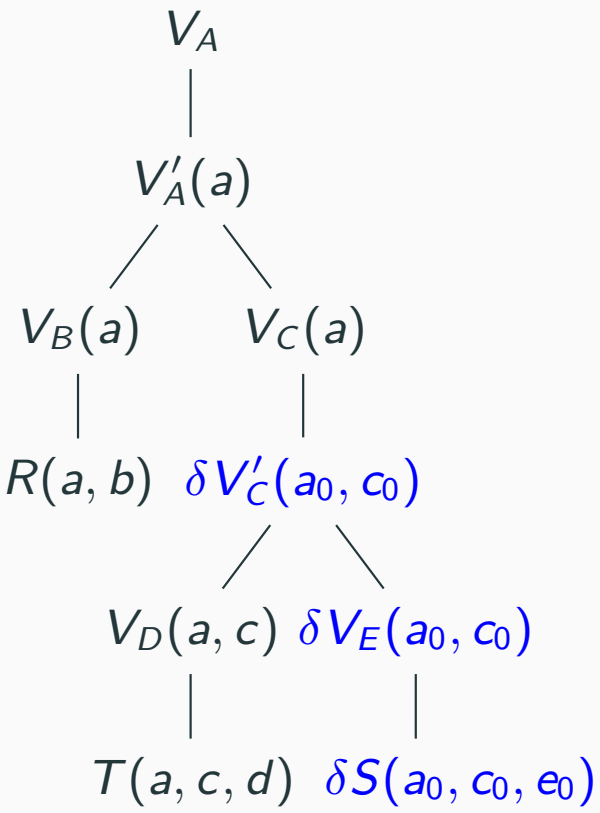

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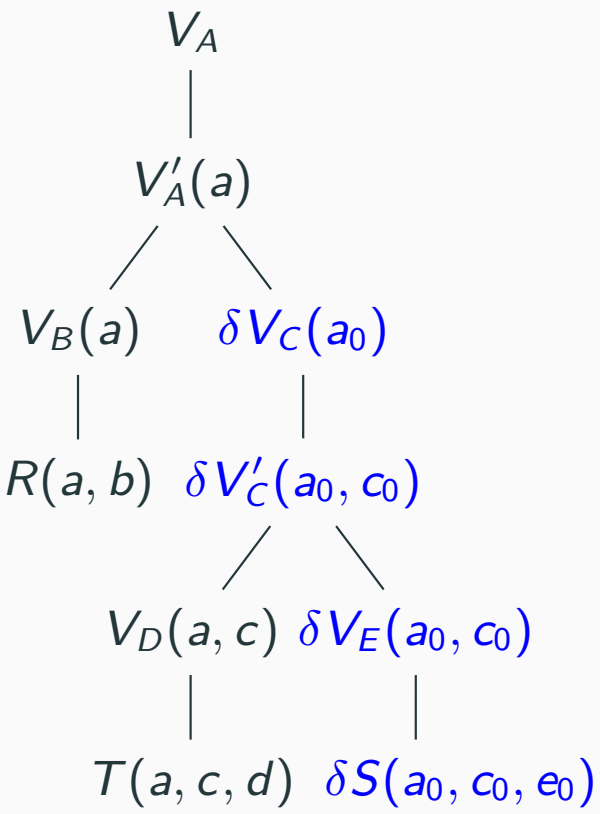

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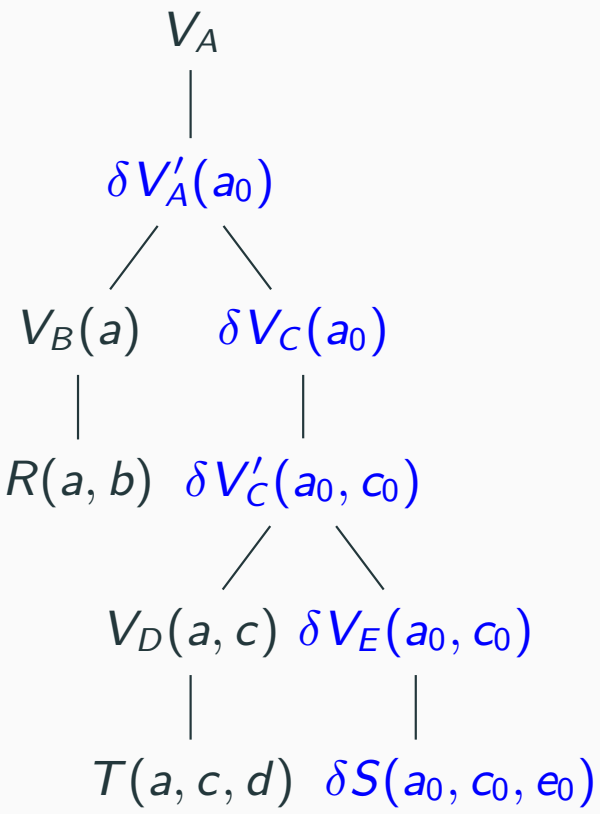

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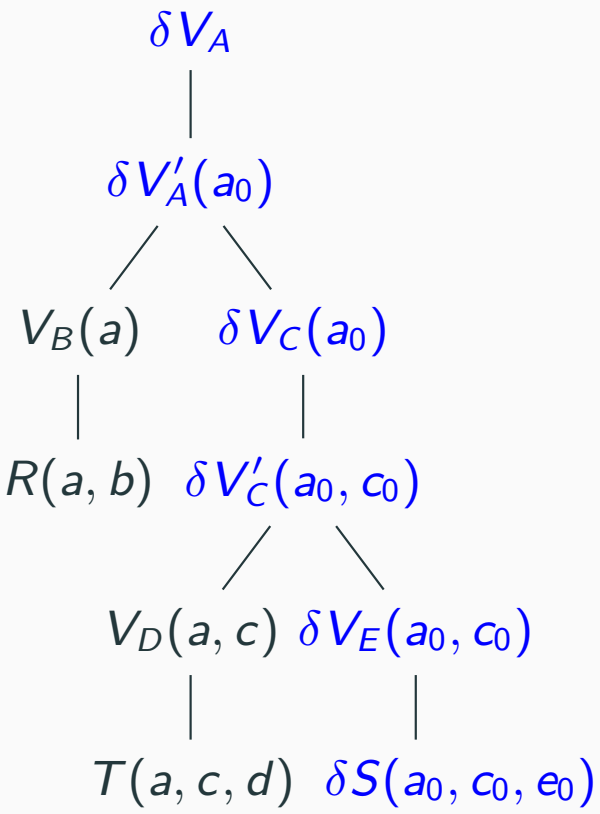

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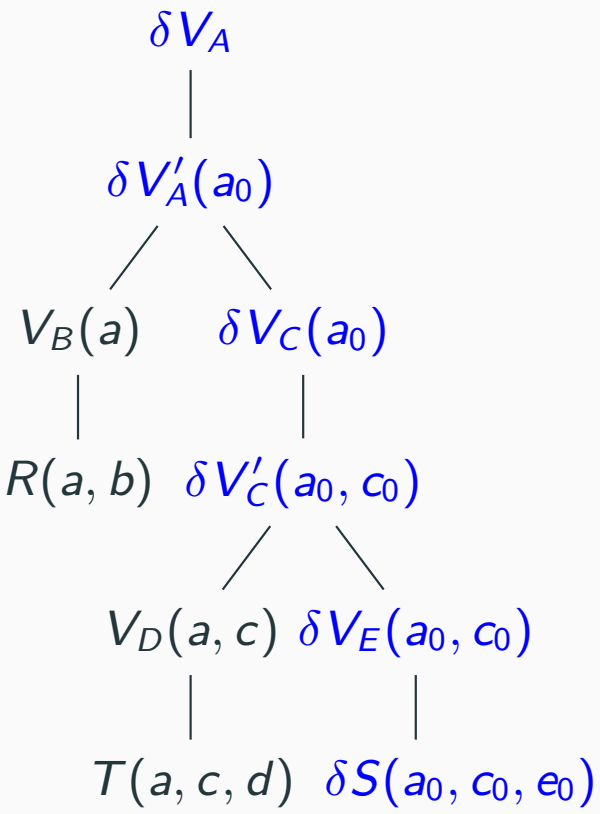

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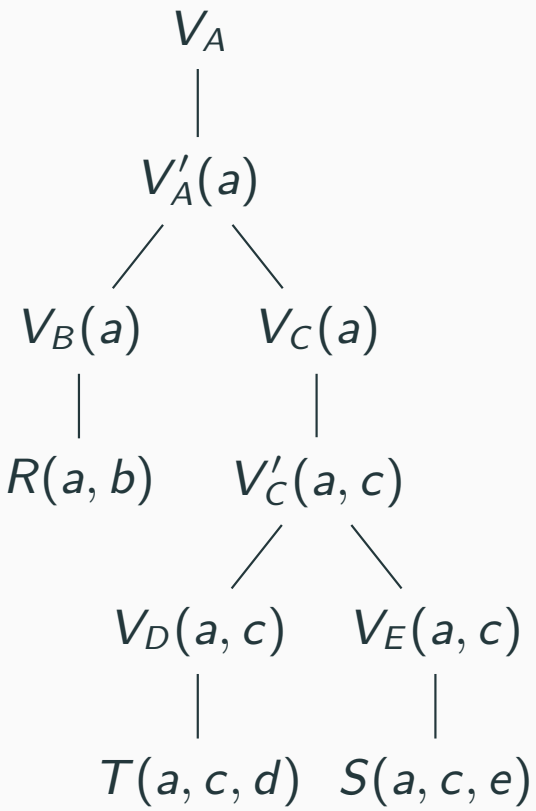

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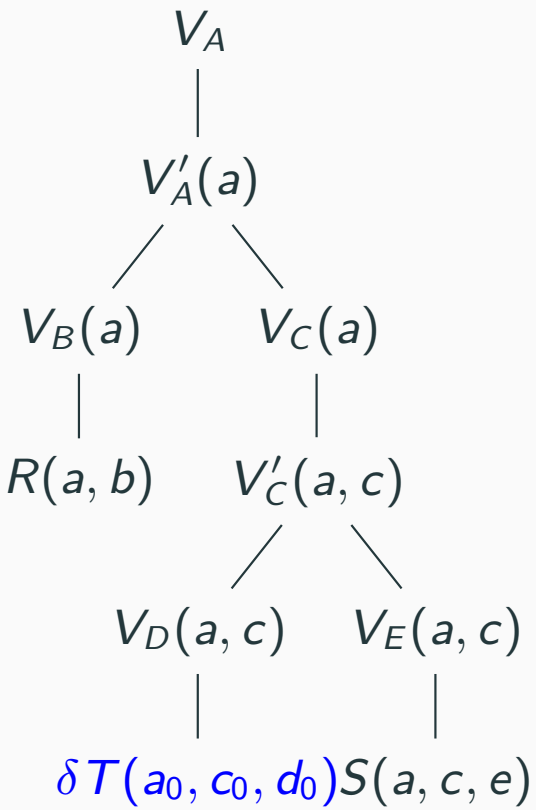

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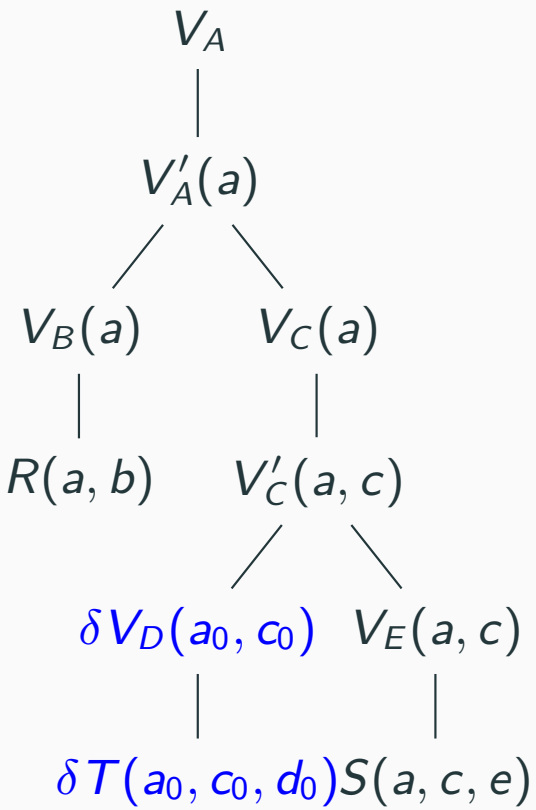

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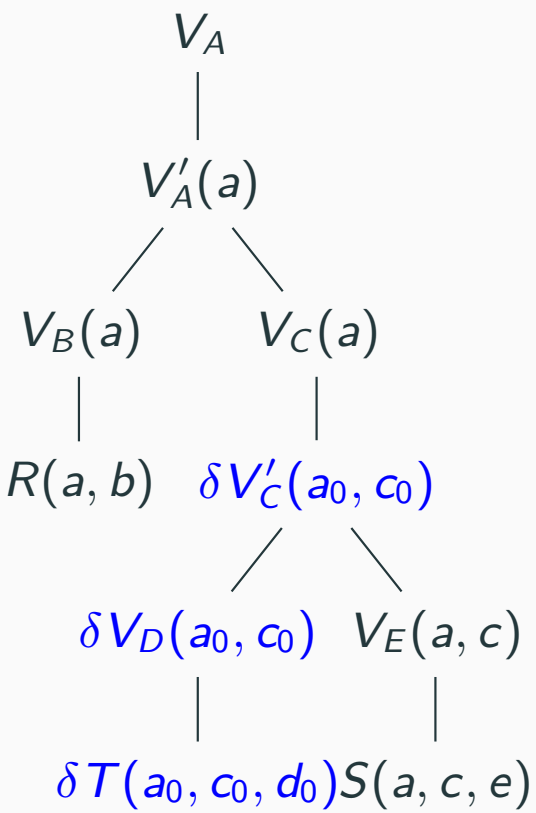

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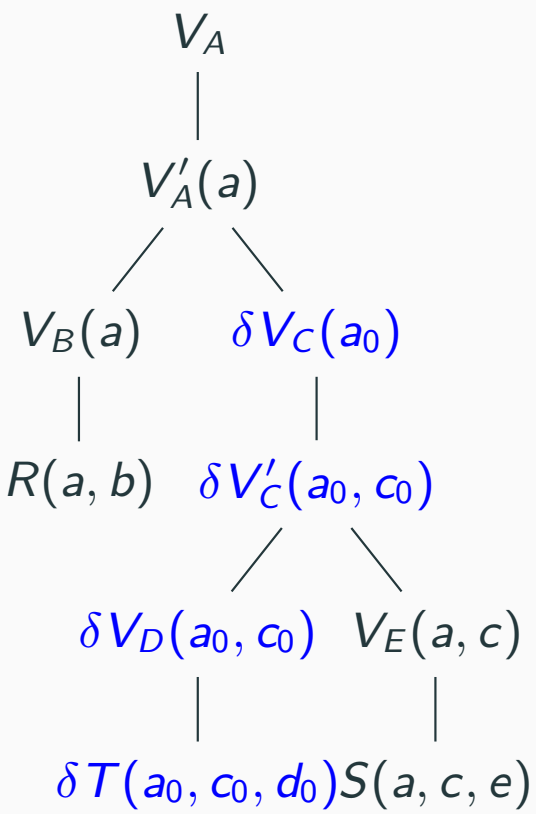

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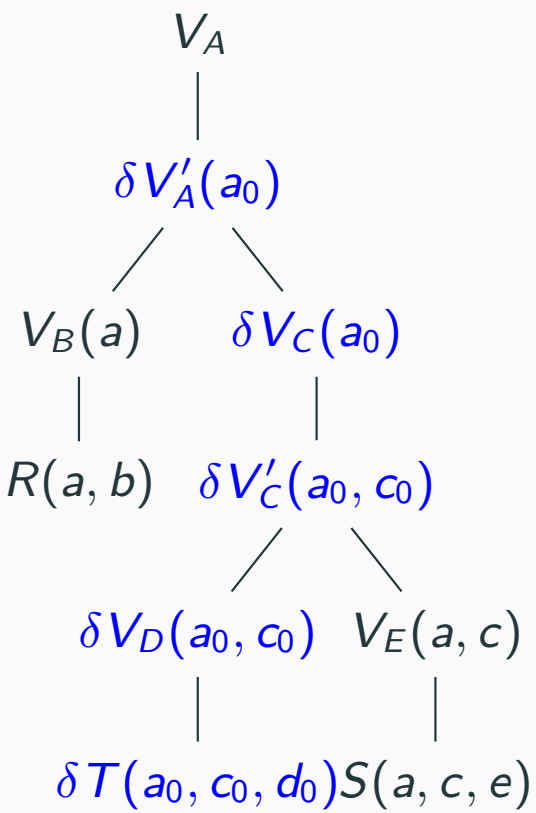

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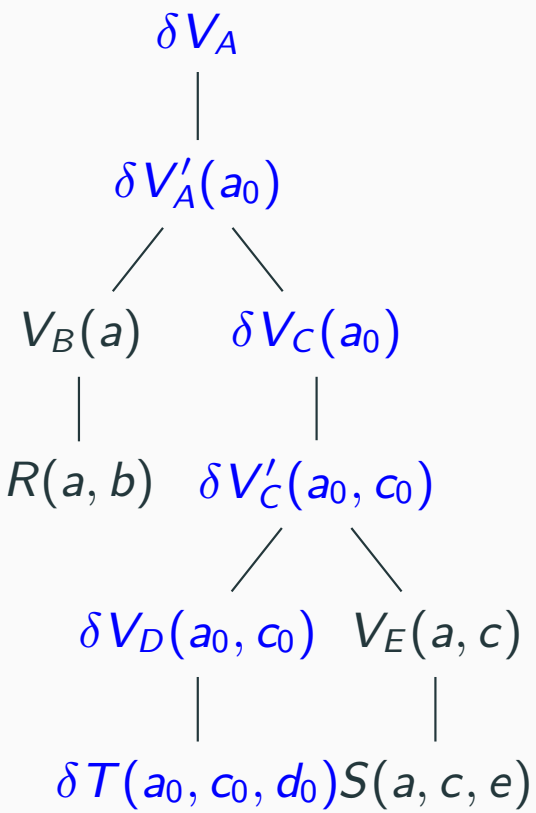

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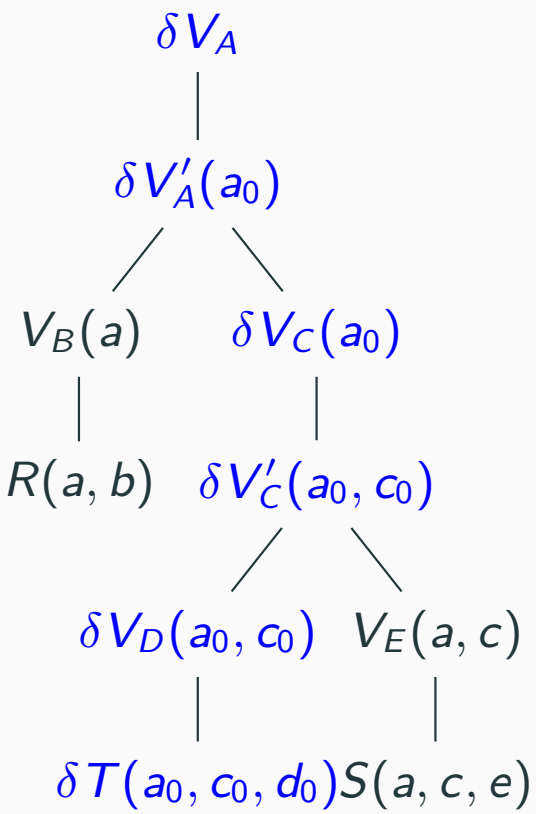

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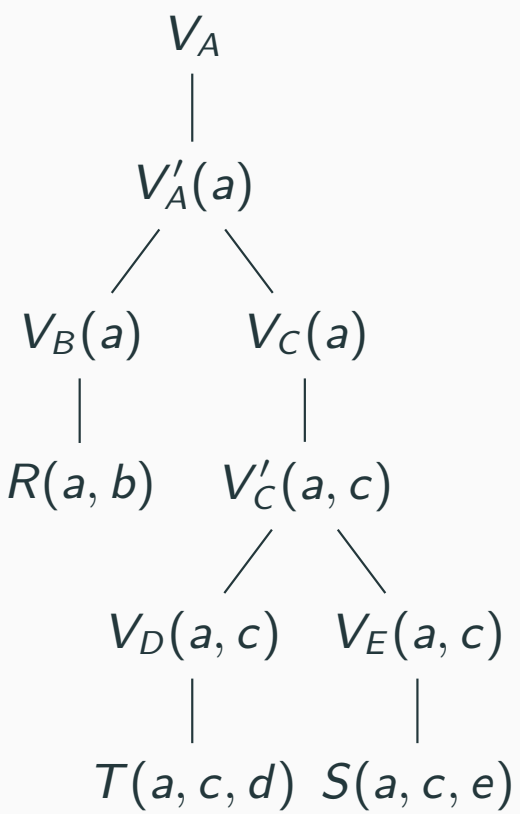

Enumeration for $Q(a,b,c)$ with constant delay

- Top-down in the view tree
- Views calibrated for variables underneath
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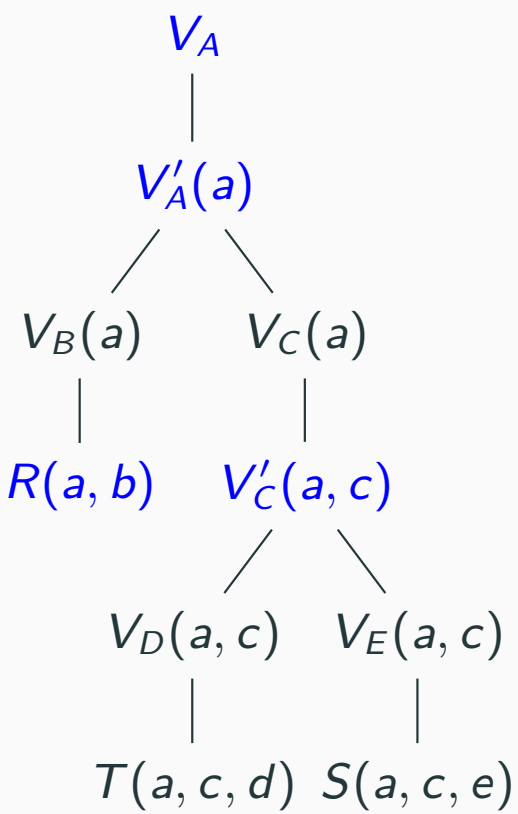

Enumeration for $Q(a,b,c)$ with constant delay

- Top-down in the view tree
- Views calibrated for variables underneath
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- Is V_A empty? If yes, stop.
- Iterate over a's in $V'_A(a)$
- For each a, iterate over b's in $R(a,b)$

  Requires index $a \mapsto b$ in R
- For each a, b, iterate over c's in $V'_C(a,c)$

  Requires index $a \mapsto c$ in V'_C
- Output (a,b,c)

Yet DBToaster & F-IVM remain Sub-Optimal

DBToaster & F-IVM are not optimal

for a variety of queries,

including the triangle count

Extra Slides:

DBToaster and F-IVM for Triangle Count

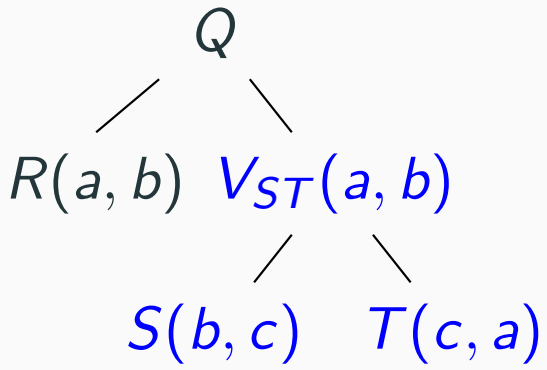
F-IVM for the Triangle Count Query (1/2)

$$Q = \sum_{a,b,c} R(a,b) \cdot S(b,c) \cdot T(c,a)$$

A single-tuple update δR to R takes time $\mathcal{O}(1)$:

F-IVM creates the materialized view V_{ST}

$$V_{ST}(a,b) = \sum_{c} S(b,c) \cdot T(c,a)$$



Computation time: $\mathcal{O}(N^2)$

F-IVM for the Triangle Count Query (1/2)

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The delta query for update δR using V_{ST} is:

$$\delta Q = \sum_{a_0,b_0} \delta R(a_0,b_0) \cdot V_{ST}(a_0,b_0)$$

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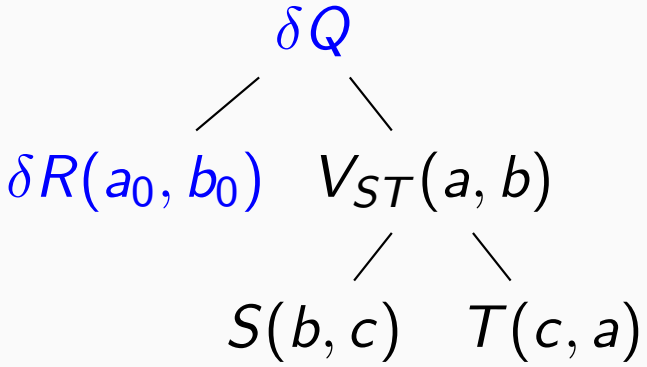

F-IVM for the Triangle Count Query (2/2)

$$Q = \sum_{a,b,c} R(a,b) \cdot S(b,c) \cdot T(c,a)$$

Q and the view V_{ST} need $\mathcal{O}(N)$ for single-tuple updates:

Delta queries for updates δS and δT:

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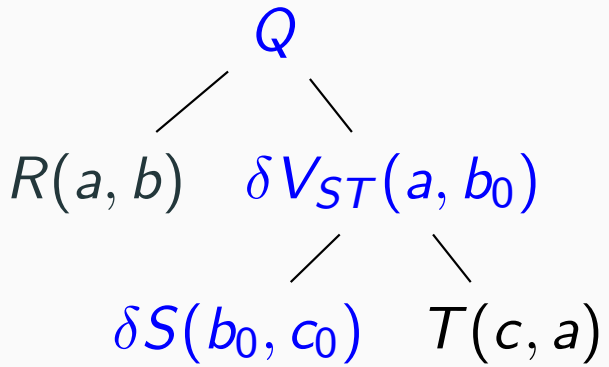

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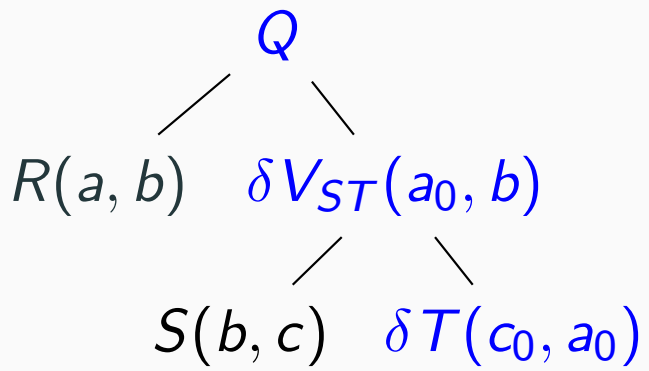
$$\delta V_{ST}(a, b_0) = \delta S(b_0, c_0) \cdot T(c_0, a)$$

$$\delta Q = \delta V_{ST}(a, b_0) \cdot R(a, b_0)$$

$$\delta V_{ST}(a_0, b) = \delta T(c_0, a_0) \cdot S(b, c_0)$$

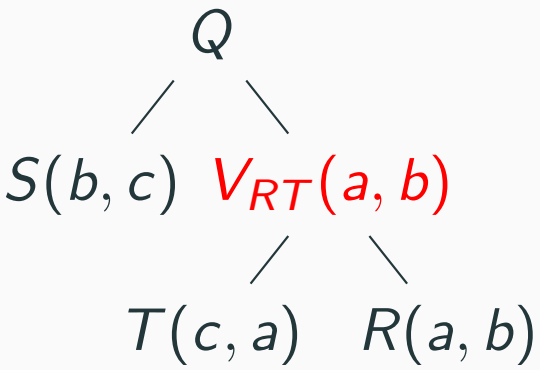
$$\delta Q = \delta V_{ST}(a_0, b) \cdot R(a_0, b)$$

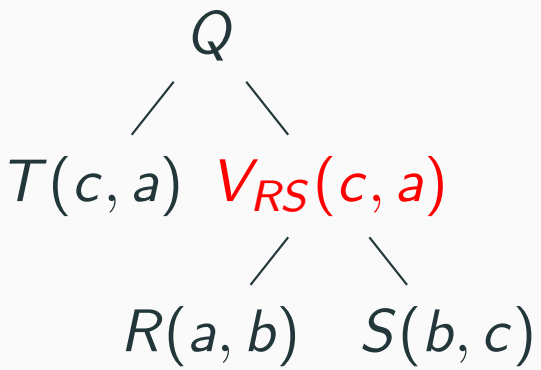


DBToaster for the Triangle Count Query

$$Q = \sum_{a,b,c} R(a,b) \cdot S(b,c) \cdot T(c,a)$$

DBToaster also creates the materialized views V_{RT} and V_{RS} to support updates to S and T:





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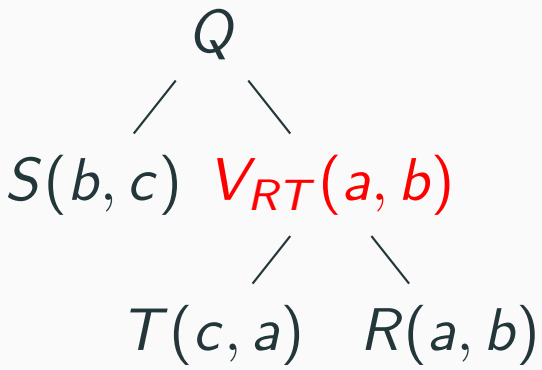

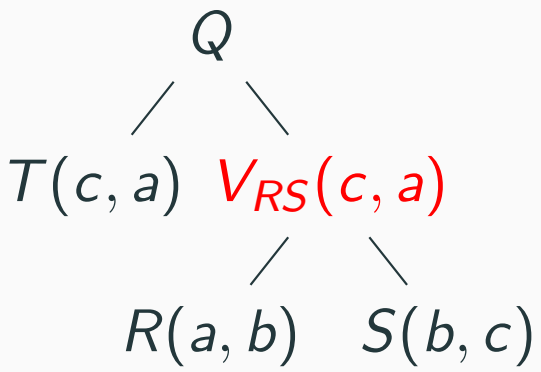

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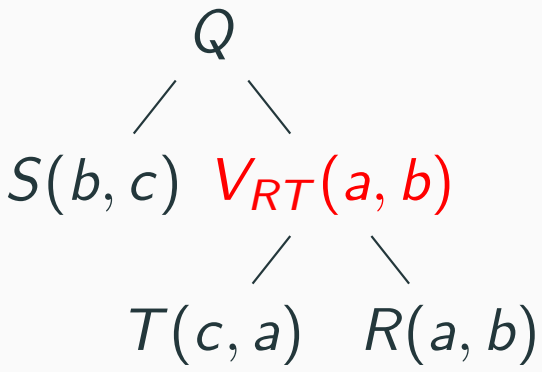

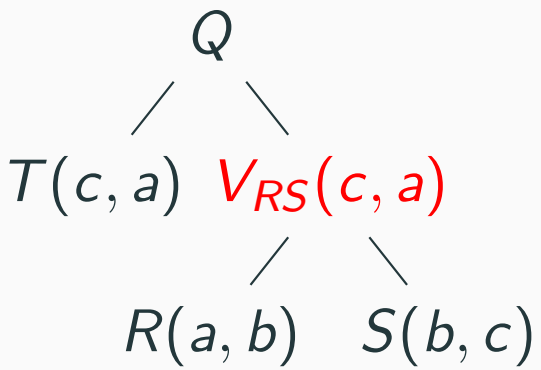

They again require $\mathcal{O}(N)$ update time

Single-tuple updates to each of the three factors require $\mathcal{O}(N)$ time

IVM for the Triangle Count Query: Summing Up

$$Q = \sum_{a,b,c} R(a,b) \cdot S(b,c) \cdot T(c,a)$$

- First-order IVM: $\mathcal{O}(N)$ time for update to any factor

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- Higher-order IVM using DBToaster
  - creates three view trees
  - $\mathcal{O}(N)$ time for update to any factor

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- Higher-order IVM using F-IVM
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Can we achieve the optimal update time of $\mathcal{O}(N^{1/2})$?

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Can we achieve the optimal update time of $\mathcal{O}(N^{1/2})$?

Yes! Adaptive IVM

Extra Slides End

Part 2.

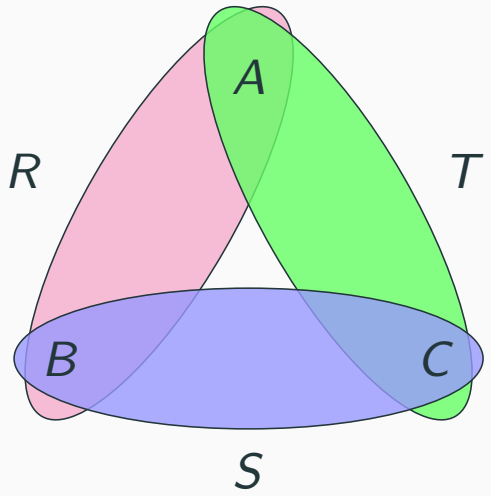
Main IVM techniques:

Adaptive IVM

The Triangle Count Query

The triangle count query Q returns the number of tuples in the join of R, S, and T:

$$Q = \sum_{a,b,c} R(a,b) \cdot S(b,c) \cdot T(c,a)$$



Problem: Maintain Q under single-tuple updates to R, S, and T

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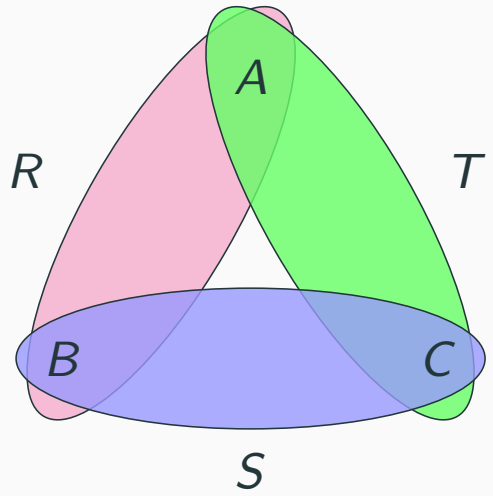

Problem: Maintain Q under single-tuple updates to R, S, and T

Is there a **fully dynamic algorithm** that can maintain the **exact triangle count** in **worst-case optimal** time?

Naïve Maintenance

"*Recompute from scratch*"

$$Q = \sum_{a,b,c} R(a,b) \cdot S(b,c) \cdot T(c,a)$$

Naïve Maintenance

"*Recompute from scratch*"

$$\delta R = \{(\alpha, \beta) \mapsto m\}$$

$$Q = \sum_{a,b,c} \underbrace{\boxed{}}_{R(a,b)+\delta R(a,b)} \cdot \underbrace{\boxed{}}_{S(b,c)} \cdot \underbrace{\boxed{}}_{T(c,a)}$$

Naïve Maintenance

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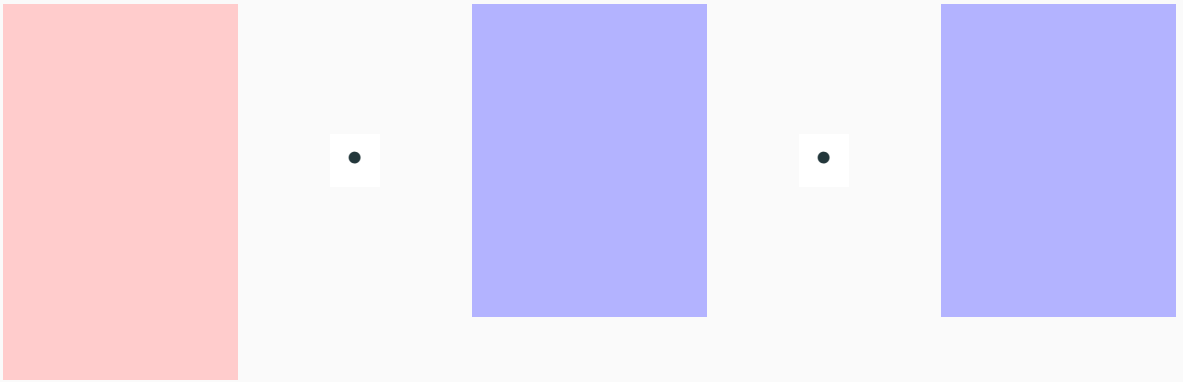
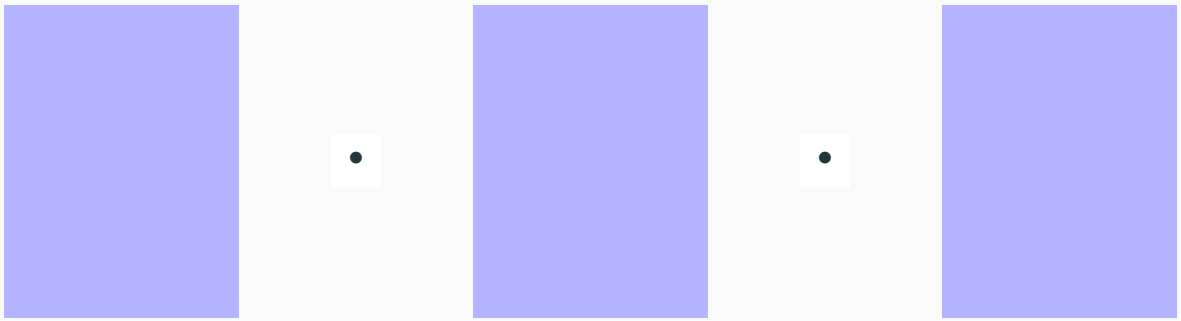


- N is the database size
- Update time: $\mathcal{O}(N^{1.5})$ using worst-case optimal join algorithms
  [Algorithmica 1997, SIGMOD R. 2013, ICDT 2014]
  Slightly better using Strassen-like matrix multiplication
- Space: $\mathcal{O}(N)$ to store input factors

First-Order Incremental View Maintenance

"Compute the delta" *[Found. & Trends DB 2018]*

$$Q = \sum_{a,b,c} R(a,b) \cdot S(b,c) \cdot T(c,a)$$



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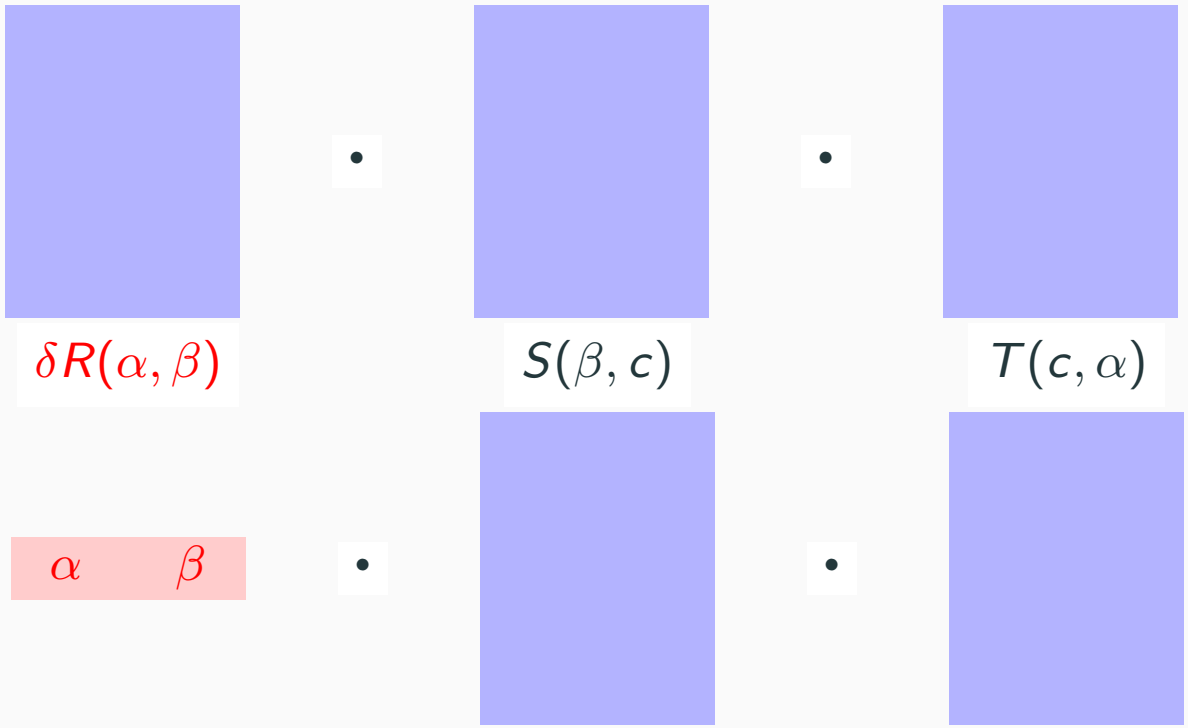

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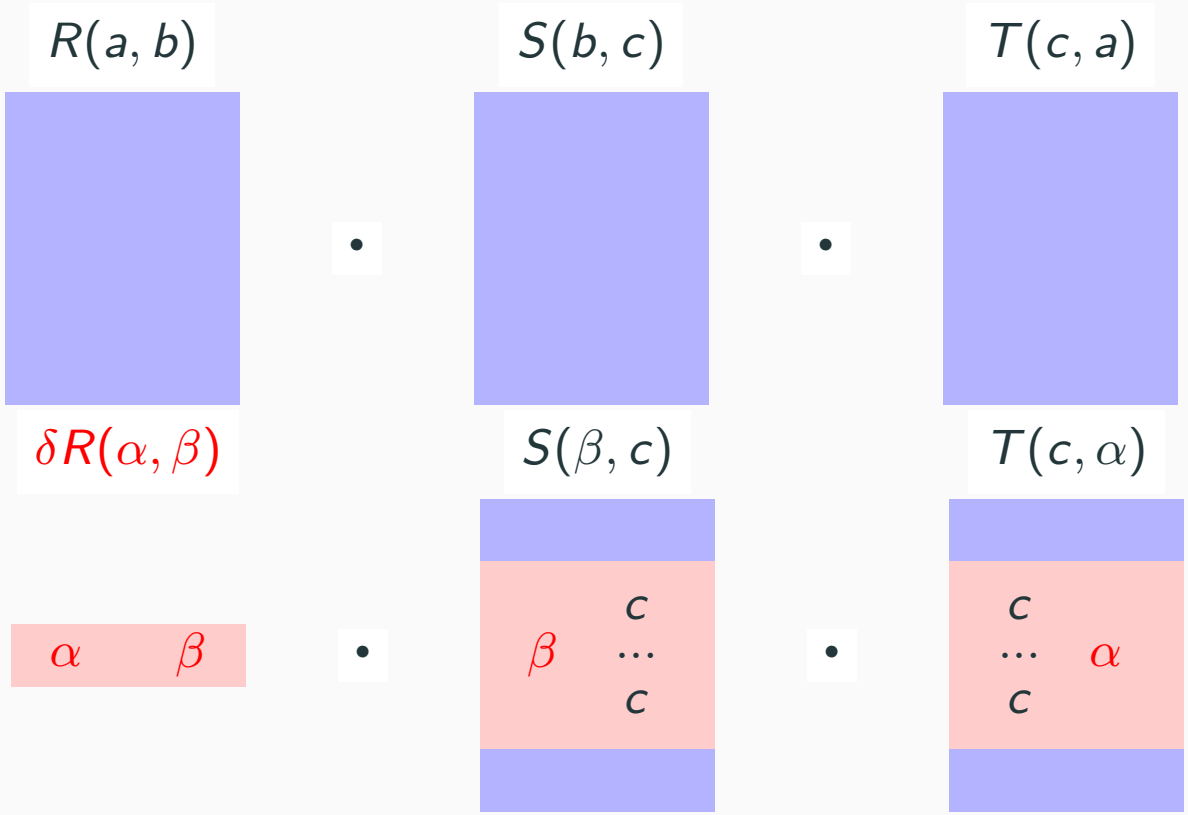

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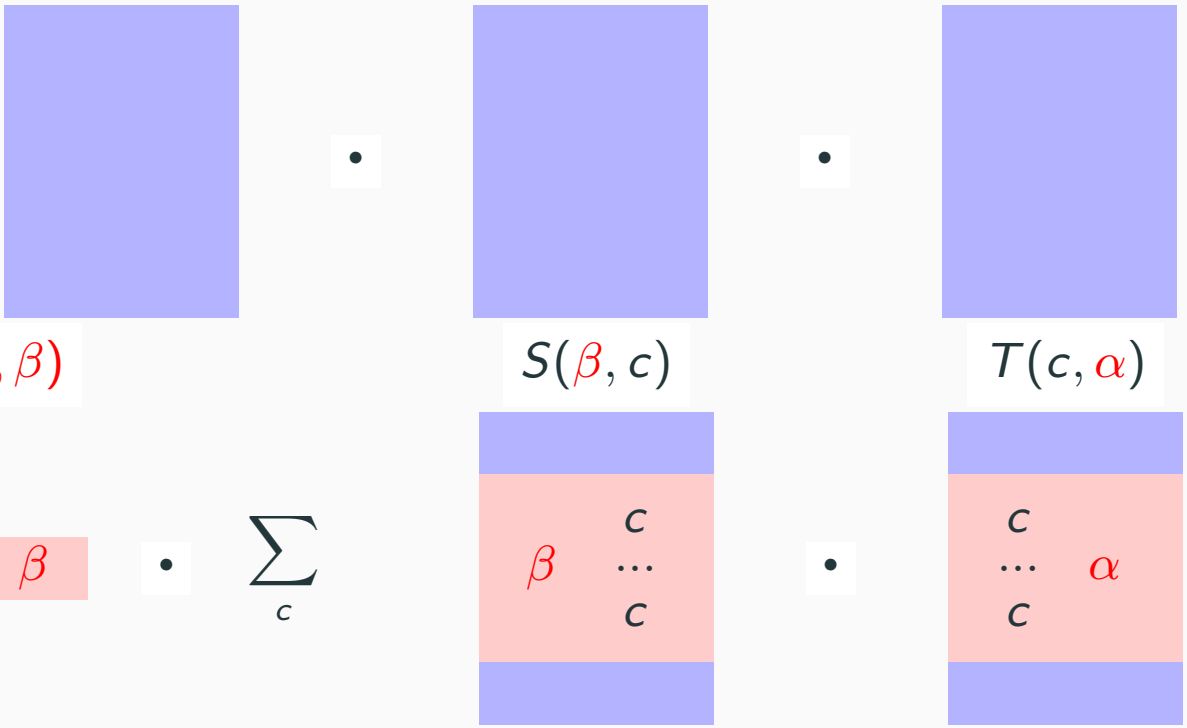

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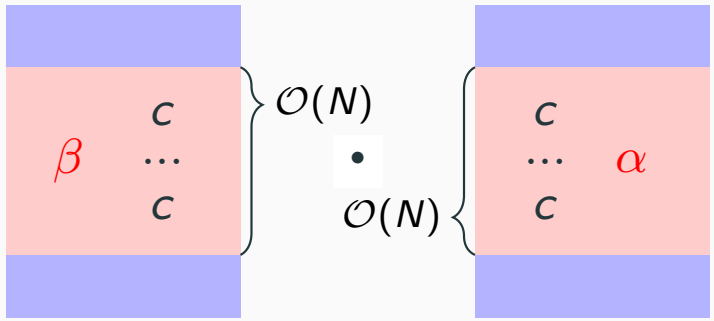

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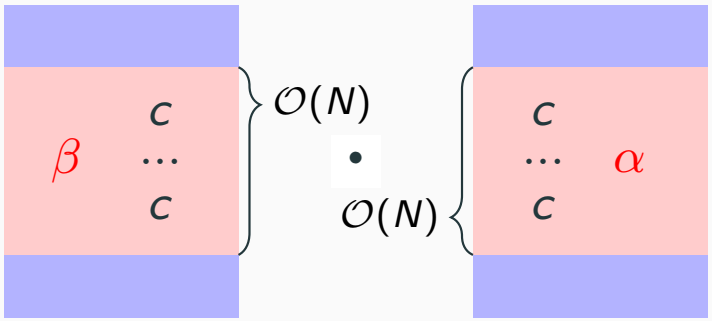

$$Q = Q + \delta Q$$

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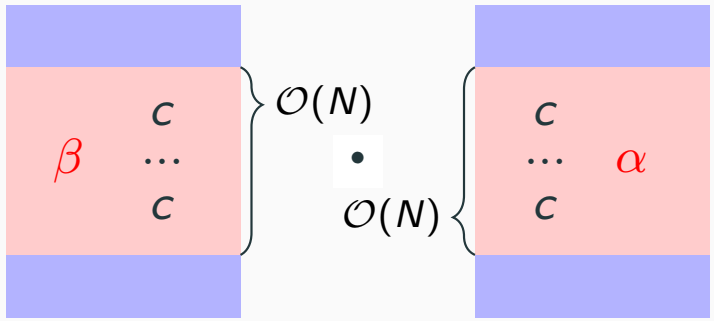

$$Q = Q + \delta Q$$

- Update time: $\mathcal{O}(N)$ to intersect C-values from S and T
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Higher-Order Incremental View Maintenance

"*Compute the delta using materialized views*" *[VLDB J 2014]*

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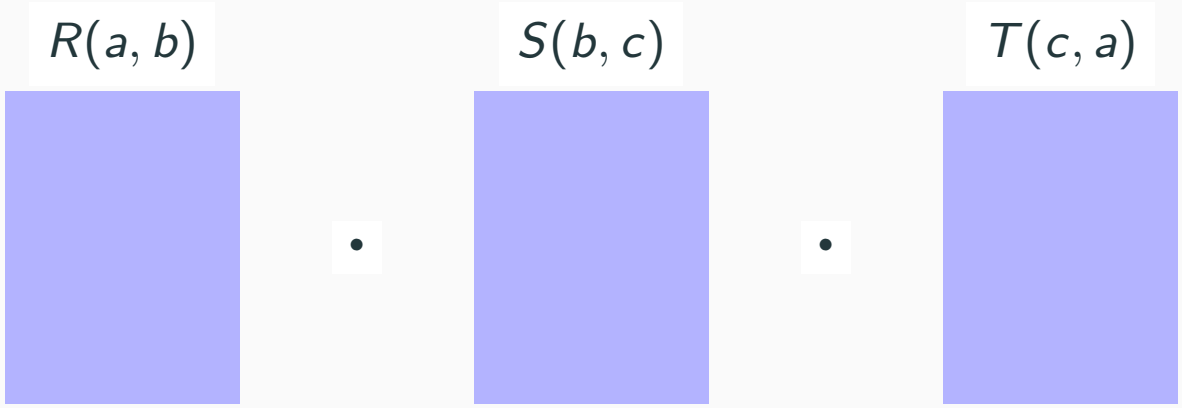

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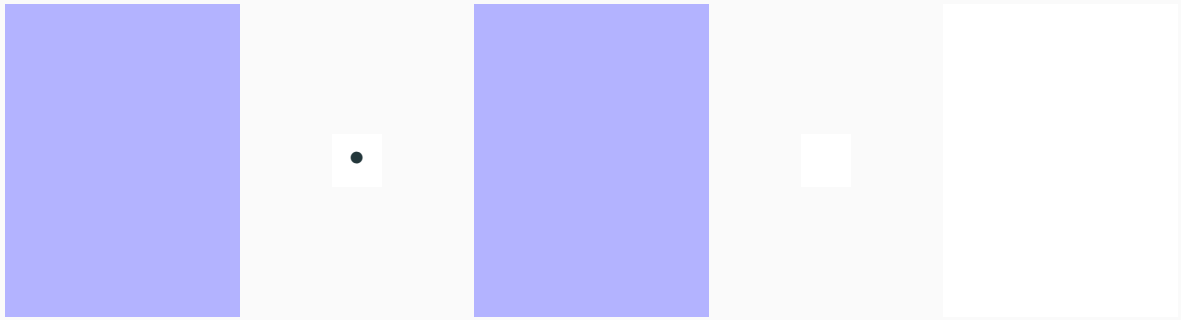

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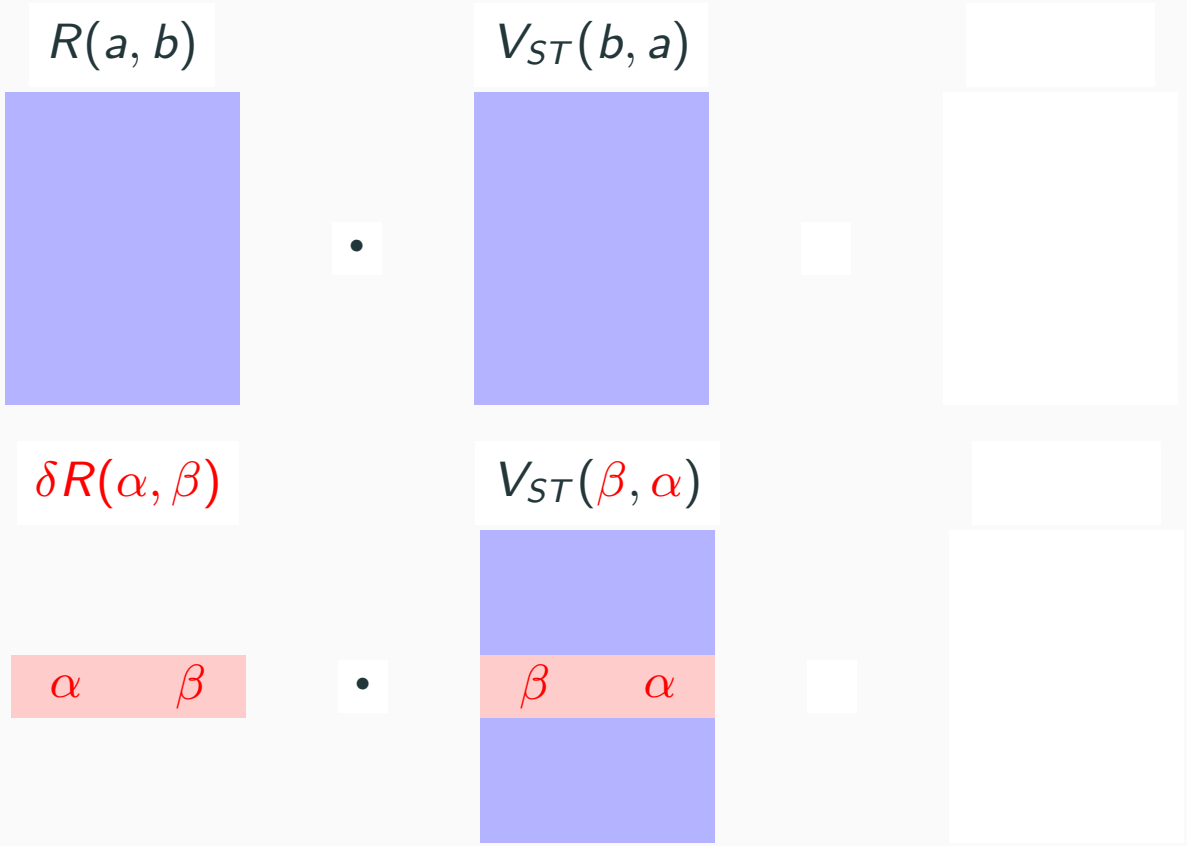

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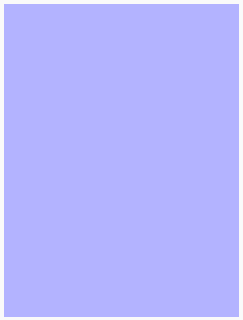

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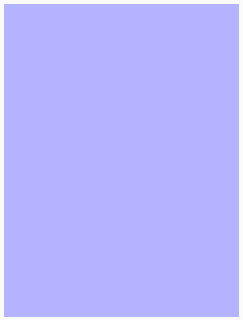

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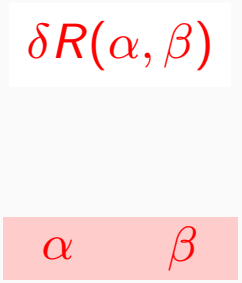

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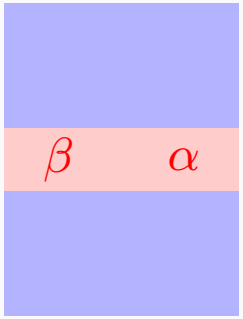

$$Q = Q + \delta Q$$

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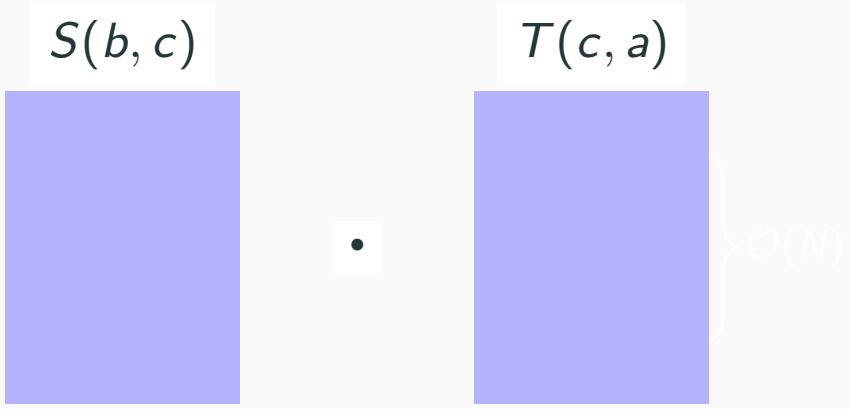
$$Q = Q + \delta Q$$

- Time for updates to R: $\mathcal{O}(1)$ to look up in V_{ST}

Higher-Order Incremental View Maintenance

Maintain V_{ST} under updates

$$V_{ST}(b, a) = \sum_{c}$$

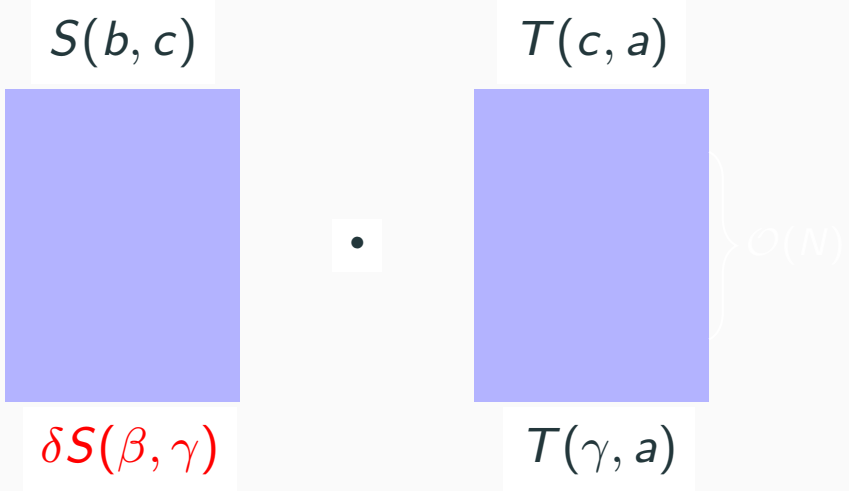


Higher-Order Incremental View Maintenance

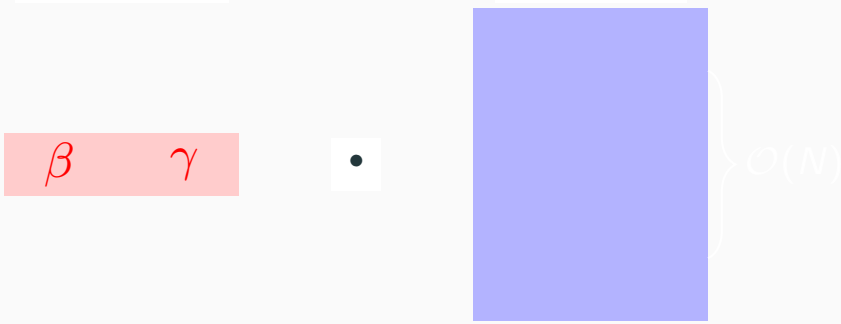
Maintain V_{ST} under updates

$$\delta S = \{(\beta, \gamma) \mapsto m\}$$

$$V_{ST}(b, a) = \sum_c$$



$$\delta V_{ST}(\beta, a) =$$



Higher-Order Incremental View Maintenance

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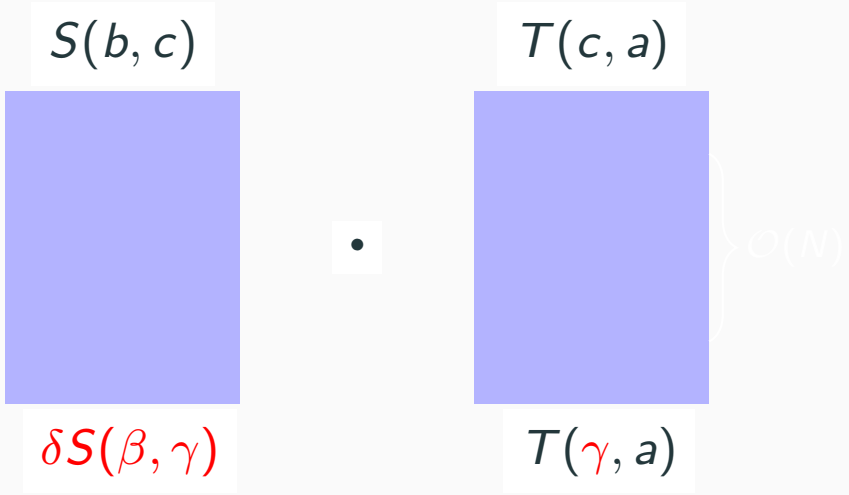

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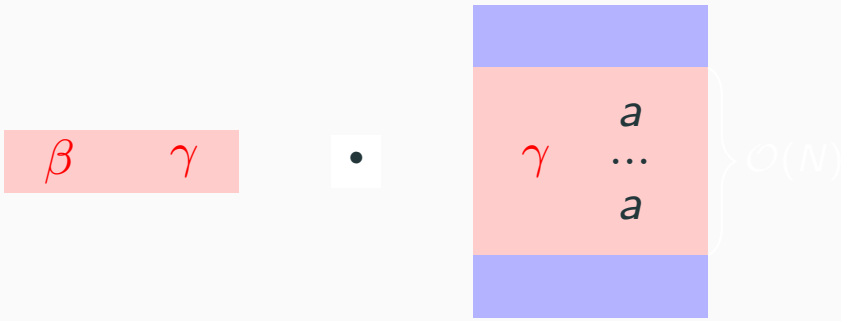
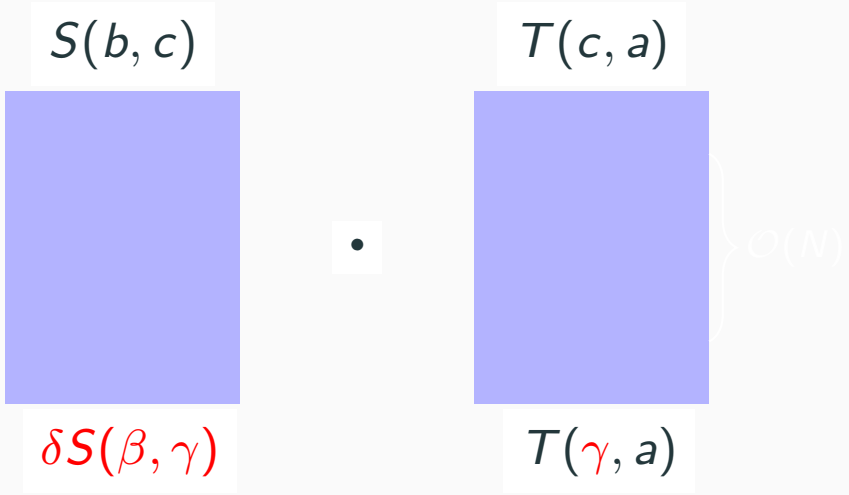


Higher-Order Incremental View Maintenance

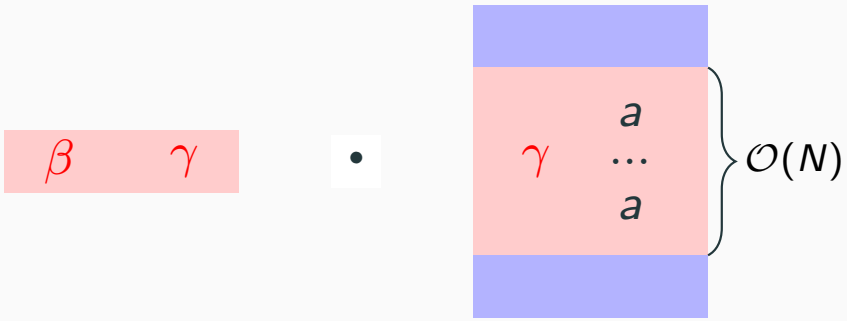
Maintain V_{ST} under updates

$$\delta S = \{(\beta, \gamma) \mapsto m\}$$

$$V_{ST}(b, a) = \sum_c$$



$$\delta V_{ST}(\beta, a) =$$



Higher-Order Incremental View Maintenance

Maintain V_{ST} under updates

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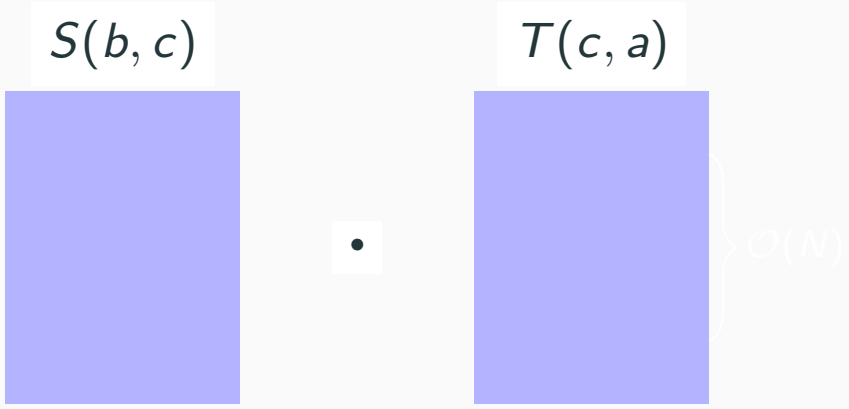

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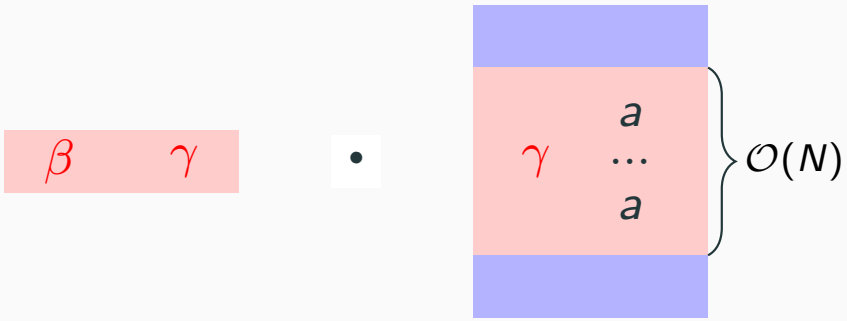

$$V_{ST}(\beta, a) = V_{ST}(\beta, a) + \delta V_{ST}(\beta, a)$$

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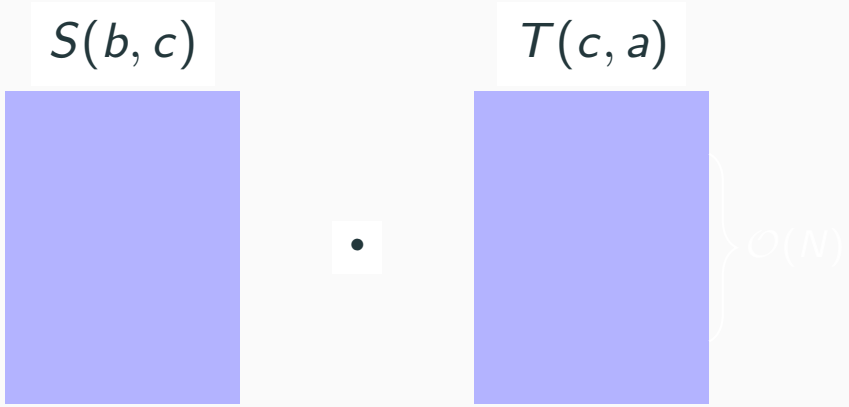

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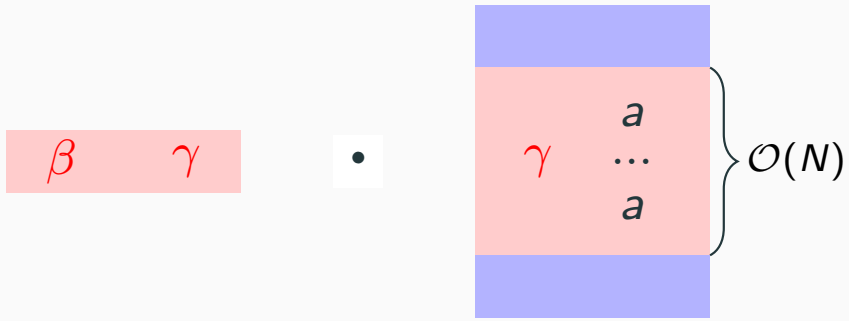

$$V_{ST}(\beta, a) = V_{ST}(\beta, a) + \delta V_{ST}(\beta, a)$$

- Time for updates to S and T: $\mathcal{O}(N)$ to maintain V_{ST}
- Space: $\mathcal{O}(N^2)$ to store input factors and V_{ST}

Lower Bound for Maintaining the Triangle Count

The Boolean Triangle Detection Problem

Boolean Triangle Detection Query

$$Q_b = \bigvee_{a,b,c} R(a,b) \wedge S(b,c) \wedge T(c,a)$$

The Boolean Triangle Detection Problem

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$$Q_b = \bigvee_{a,b,c} R(a,b) \wedge S(b,c) \wedge T(c,a)$$

Let **D** be the database instance and N the number of tuples in **D**.
For any $\gamma > 0$, there is no algorithm that incrementally maintains Q_b with

update time	**and**	enumeration delay
$\mathcal{O}(N^{\frac{1}{2}-\gamma})$		$\mathcal{O}(N^{1-\gamma})$

unless the Online Vector-Matrix-Vector Multiplication (OuMv) Conjecture fails.

Online Vector-Matrix-Vector Multiplication

The OuMv problem:

- Input: An $n \times n$ Boolean matrix $\mathbf{M}$ and n pairs $(\mathbf{u}_1, \mathbf{v}_1), \ldots, (\mathbf{u}_n, \mathbf{v}_n)$ of Boolean column-vectors of size n arriving one after the other.
- Goal: After seeing each pair $(\mathbf{u}_r, \mathbf{v}_r)$, output $\mathbf{u}_r^\top \mathbf{M} \mathbf{v}_r$

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The OuMv Conjecture [*STOC 2015*]

For any $\gamma > 0$, there is no algorithm that solves OuMv in time $\mathcal{O}(n^{3-\gamma})$.

Proof Idea

- Assume there is an algorithm $\mathcal{A}$ that can maintain Triangle Detection Query Q_b with

amortized update time	**and**	enumeration delay
$\mathcal{O}(N^{\frac{1}{2}-\gamma})$		$\mathcal{O}(N^{1-\gamma})$

  for some $\gamma > 0$.

- We design an algorithm $\mathcal{B}$ that uses the oracle $\mathcal{A}$ to solve OuMv in subcubic time in n. $\Longrightarrow$ **Contradicts the OuMv Conjecture!**

Proof Idea

- Assume there is an algorithm $\mathcal{A}$ that can maintain Triangle Detection Query Q_b with

  amortized update time $\mathcal{O}(N^{\frac{1}{2}-\gamma})$ **and** enumeration delay $\mathcal{O}(N^{1-\gamma})$

  for some $\gamma > 0$.

- We design an algorithm $\mathcal{B}$ that uses the oracle $\mathcal{A}$ to solve OuMv in subcubic time in n. $\Longrightarrow$ **Contradicts the OuMv Conjecture!**

Algorithm $\mathcal{B}$

- Factor S encodes the matrix $\mathbf{M}$: $S(i,j) = \mathbf{M}[i,j]$
- In each round $r \in [n]$:
  - Factor R encodes the vector $\mathbf{u}_r$: $R(\mathrm{a}, i) = \mathbf{u}_r[i]$, for constant a
  - Factor T encodes the vector $\mathbf{v}_r$: $T(j, \mathrm{a}) = \mathbf{v}_r[j]$, for constant a
  - Then $\mathbf{u}_r^\top \mathbf{M} \mathbf{v}_r = Q_b$
  - Check whether $Q_b = 1$ using algorithm $\mathcal{A}$.

Example Encoding for u, M, and v

$\mathbf{u}^\top$

0	1	0

$\mathbf{M}$

0	1	0
1	1	0
1	0	1

$\mathbf{v}$

1
0
0

$\mathbf{u}^\top\mathbf{M}\mathbf{v}$

1

R

A	B	val
a	2	1

S

B	C	val
2	1	1
3	1	1
1	2	1
2	2	1
3	3	1

T

C	A	val
1	a	1

Q_b

$\emptyset$	val
( )	1

Proof Sketch: Algorithm $\mathcal{B}$

(1) For $i, j \in [n]$: $S(i, j) = \mathbf{M}[i, j]$ ($\leq n^2$ insertions)

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- Check $Q_b = 1$: This holds if and only if $\mathbf{u}_r^T \mathbf{M} \mathbf{v}_r = 1$

$$\mathbf{u}_r^T \mathbf{M} \mathbf{v}_r = 1 \Leftrightarrow \exists i, j \in [n] : \mathbf{u}_r[i] = 1, \mathbf{M}[i, j] = 1, \mathbf{v}_r[j] = 1$$

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$\mathcal{B}$ constructs a database of size $N = \mathcal{O}(n^2)$.

Proof Sketch: Time Analysis

Recall $\mathcal{A}$ needs $\mathcal{O}((n^2)^{\frac{1}{2}-\gamma})$ update time and $\mathcal{O}((n^2)^{1-\gamma})$ delay

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$$\mathcal{O}(\underbrace{(n^2)^{1-\gamma}}_{\text{delay}}) = \mathcal{O}(n^{2-2\gamma})$$

For n rounds: $\mathcal{O}(n(n^{2-2\gamma} + n^{2-2\gamma})) = \mathcal{O}(n^{3-2\gamma})$

Proof Sketch: Time Analysis

Recall $\mathcal{A}$ needs $\mathcal{O}((n^2)^{\frac{1}{2}-\gamma})$ update time and $\mathcal{O}((n^2)^{1-\gamma})$ delay

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For n rounds: $\mathcal{O}(n(n^{2-2\gamma} + n^{2-2\gamma})) = \mathcal{O}(n^{3-2\gamma})$

Overall time: $\mathcal{O}(n^{3-2\gamma} + n^{3-2\gamma}) = \mathcal{O}(n^{3-2\gamma}) \Rightarrow$ Contradicts OuMv Conjecture!

Closing the Complexity Gap

Closing the Complexity Gap

Complexity bounds for the maintenance of the triangle count

Known Upper Bound

Update Time: $\mathcal{O}(N)$

Space: $\mathcal{O}(N)$

Known Lower Bound

Update time: not $\mathcal{O}(N^{\frac{1}{2}-\gamma})$ for any $\gamma > 0$

under the OuMv Conjecture

Closing the Complexity Gap

Complexity bounds for the maintenance of the triangle count

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Update Time: $\mathcal{O}(N)$

Space: $\mathcal{O}(N)$

Can the triangle count be maintained with sublinear update time?

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Update time: not $\mathcal{O}(N^{\frac{1}{2}-\gamma})$ for any $\gamma > 0$ under the OuMv Conjecture

Closing the Complexity Gap

Complexity bounds for the maintenance of the triangle count

Known Upper Bound

Update Time: $\mathcal{O}(N)$

Space: $\mathcal{O}(N)$

Can the triangle count be maintained with sublinear update time?

Yes: IVM$^\varepsilon$

Amortized update time: $\mathcal{O}(N^{\frac{1}{2}})$

This is worst-case optimal

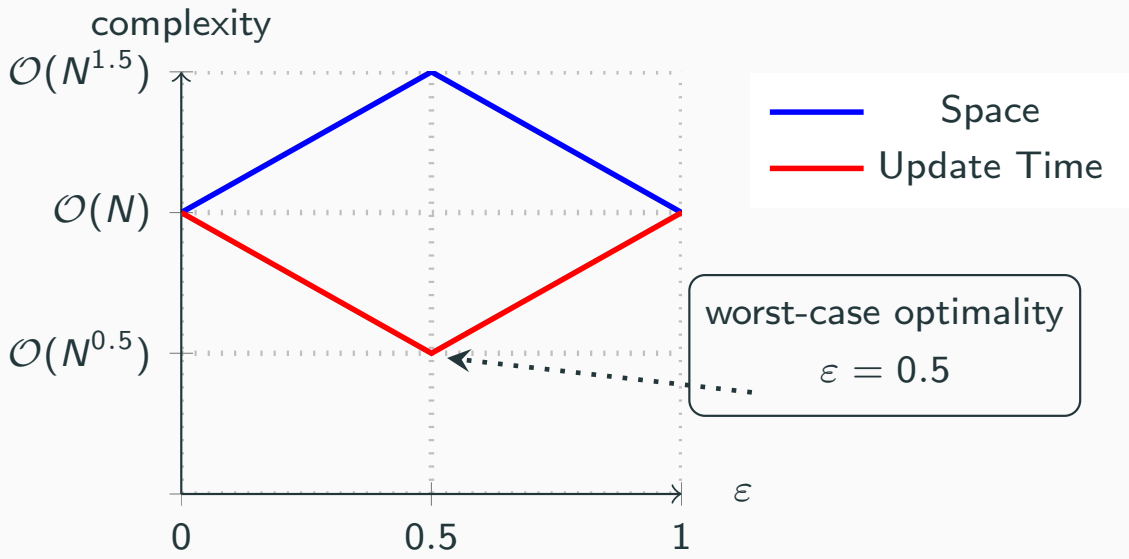
Known Lower Bound

Update time: not $\mathcal{O}(N^{\frac{1}{2}-\gamma})$ for any $\gamma > 0$ under the OuMv Conjecture

IVM$^\varepsilon$ Exhibits a Time-Space Tradeoff

Given $\varepsilon \in [0, 1]$, IVM$^\varepsilon$ maintains the triangle count with

- $\mathcal{O}(N^{\max\{\varepsilon, 1-\varepsilon\}})$ amortized update time
- $\mathcal{O}(N^{1+\min\{\varepsilon, 1-\varepsilon\}})$ space
- $\mathcal{O}(N^{\frac{3}{2}})$ preprocessing time
- $\mathcal{O}(1)$ answer time.



(Linear space possible with a slightly more involved argument)

Inside IVM$^\varepsilon$

Main Techniques used in IVM$^\varepsilon$

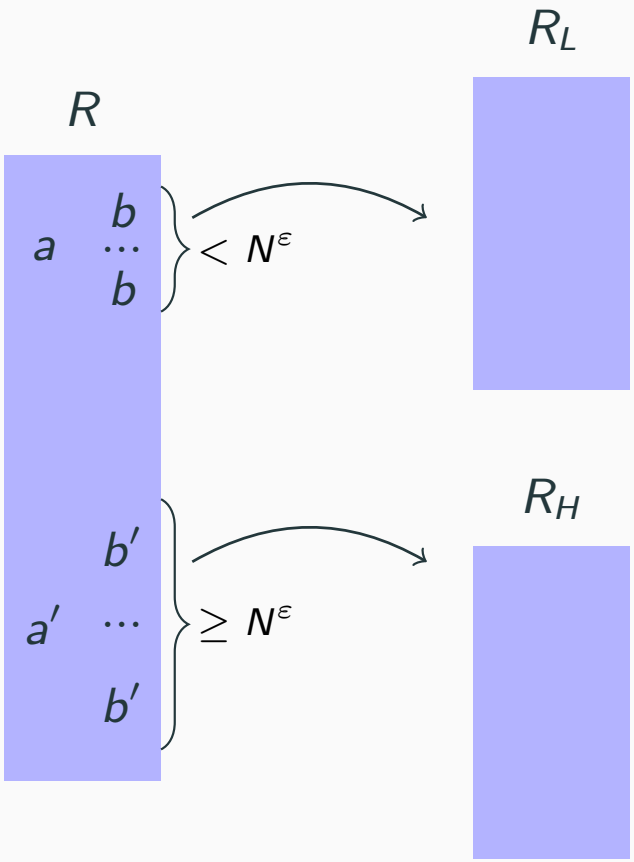
- Compute the delta like in first-order IVM
- Materialize views like in higher-order IVM
- New ingredient: Use adaptive processing based on data skew

  $\Longrightarrow$ Treat *heavy* values differently from *light* values

Heavy/Light Partitioning of Factors

Partition R based on A into

- a light part $R_L = \{t \in R \mid |\sigma_{A=t.A}| < N^\varepsilon\}$,
- a heavy part $R_H = R \backslash R_L$!



Derived Bounds

from light part:
for all A-values a, $|\sigma_{A=a} R_L| < N^\varepsilon$

from heavy part:
for all A-values a, $|\sigma_{A=a} R_H| \geq N^\varepsilon$
and $|\pi_A R_H| \cdot N^\varepsilon \leq N$
$\Longrightarrow |\pi_A R_H| \leq N^{1-\varepsilon}$

Heavy/Light Partitioning of Factors

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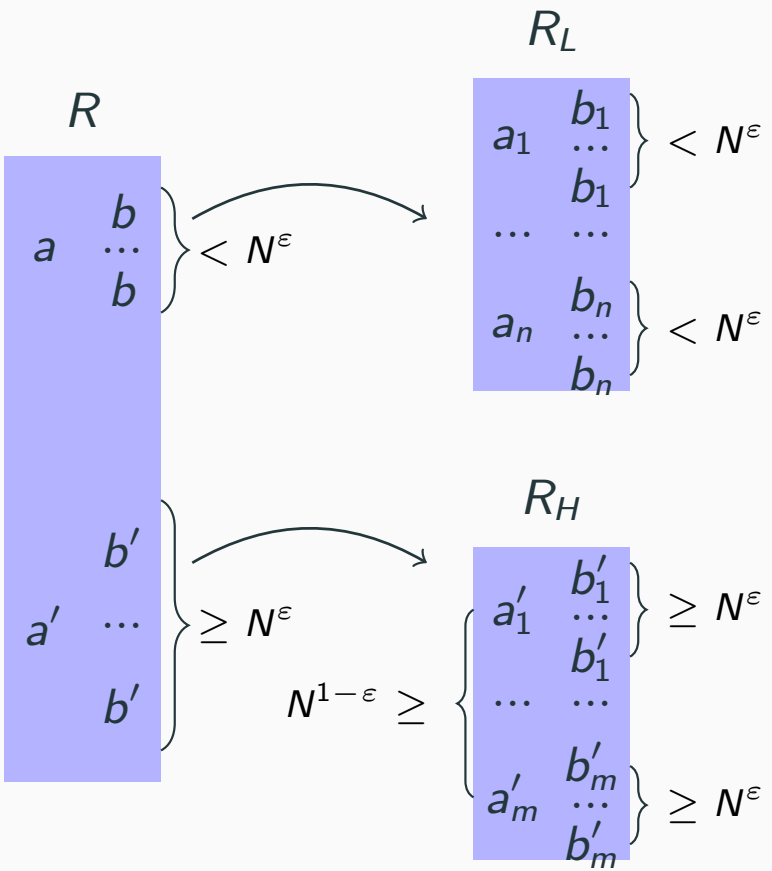

Derived Bounds

from light part:
for all A-values a, $|\sigma_{A=a} R_L| < N^\varepsilon$

from heavy part:
$|\pi_A R_H| \leq N^{1-\varepsilon}$, since
for all A-values a, $|\sigma_{A=a} R_H| \geq N^\varepsilon$
and $|\pi_A R_H| \cdot N^\varepsilon \leq N$

Heavy/Light Partitioning of Factors

Likewise, partition

- $S = S_L \cup S_H$ based on B, and
- $T = T_L \cup T_H$ based on C!

Q is the **sum** of skew-aware queries

$$Q = \sum_{a,b,c} R_U(a,b) \cdot S_V(b,c) \cdot T_W(c,a), \text{ for } U, V, W \in \{L, H\}.$$

Adaptive Maintenance Strategy

Given an update $\delta R_* = \{(\alpha, \beta) \mapsto m\}$, compute the delta for each of the following skew-aware queries using a different strategy:

$$Q_{*LL} = \sum_{a,b,c} R_*(a,b) \cdot S_L(b,c) \cdot T_L(c,a)$$

$$Q_{*HH} = \sum_{a,b,c} R_*(a,b) \cdot S_H(b,c) \cdot T_H(c,a)$$

$$Q_{*LH} = \sum_{a,b,c} R_*(a,b) \cdot S_L(b,c) \cdot T_H(c,a)$$

$$Q_{*HL} = \sum_{a,b,c} R_*(a,b) \cdot S_H(b,c) \cdot T_L(c,a)$$

Adaptive Maintenance Strategy

Given an update $\delta R_* = \{(\alpha, \beta) \mapsto m\}$, compute the delta for each of the following skew-aware queries using a different strategy:

$$\delta Q_{*LL} = \delta R_*(\alpha, \beta) \cdot \sum_c S_L(\beta, c) \cdot T_L(c, \alpha)$$

$$\delta Q_{*HH} = \delta R_*(\alpha, \beta) \cdot \sum_c S_H(\beta, c) \cdot T_H(c, \alpha)$$

$$\delta Q_{*LH} = \delta R_*(\alpha, \beta) \cdot \sum_c S_L(\beta, c) \cdot T_H(c, \alpha)$$

$$\delta Q_{*HL} = \delta R_*(\alpha, \beta) \cdot \sum_c S_H(\beta, c) \cdot T_L(c, \alpha)$$

Adaptive Maintenance Strategy

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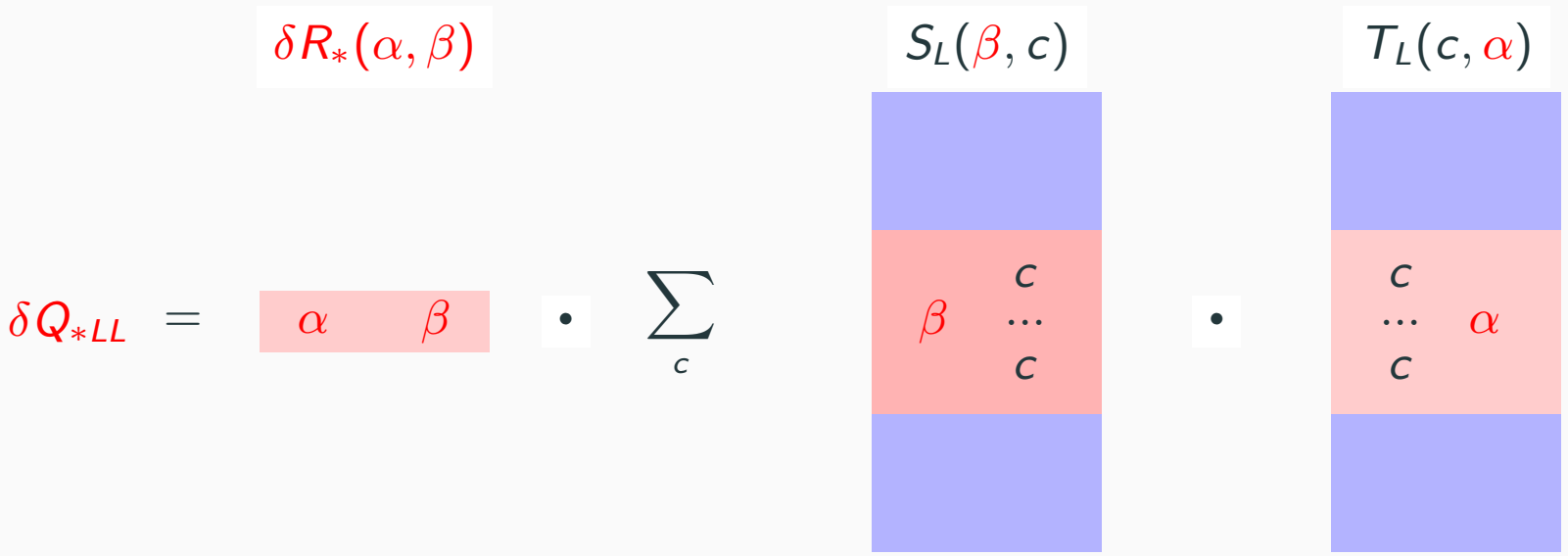

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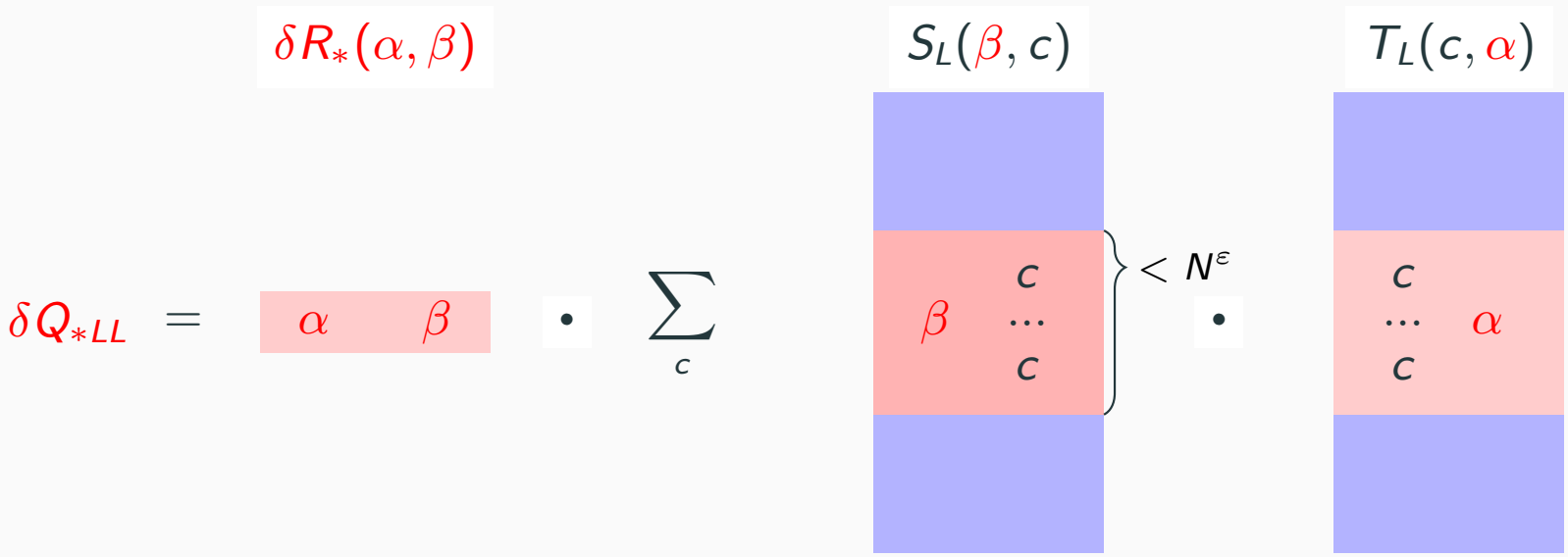

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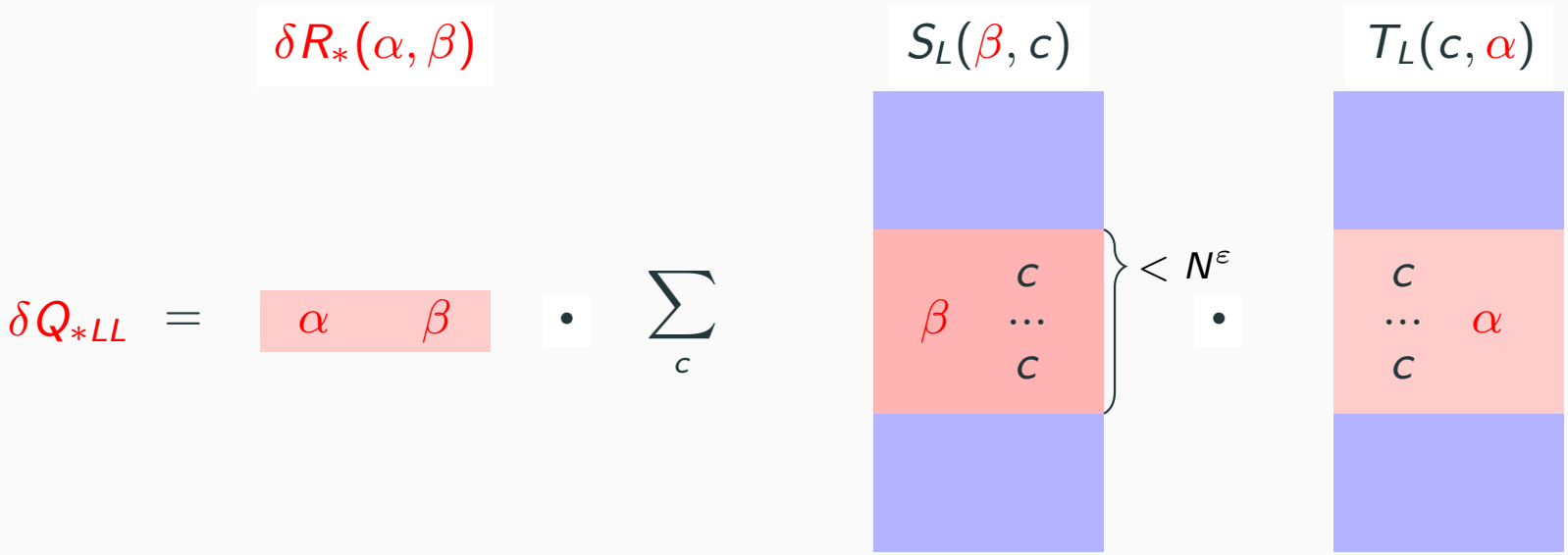
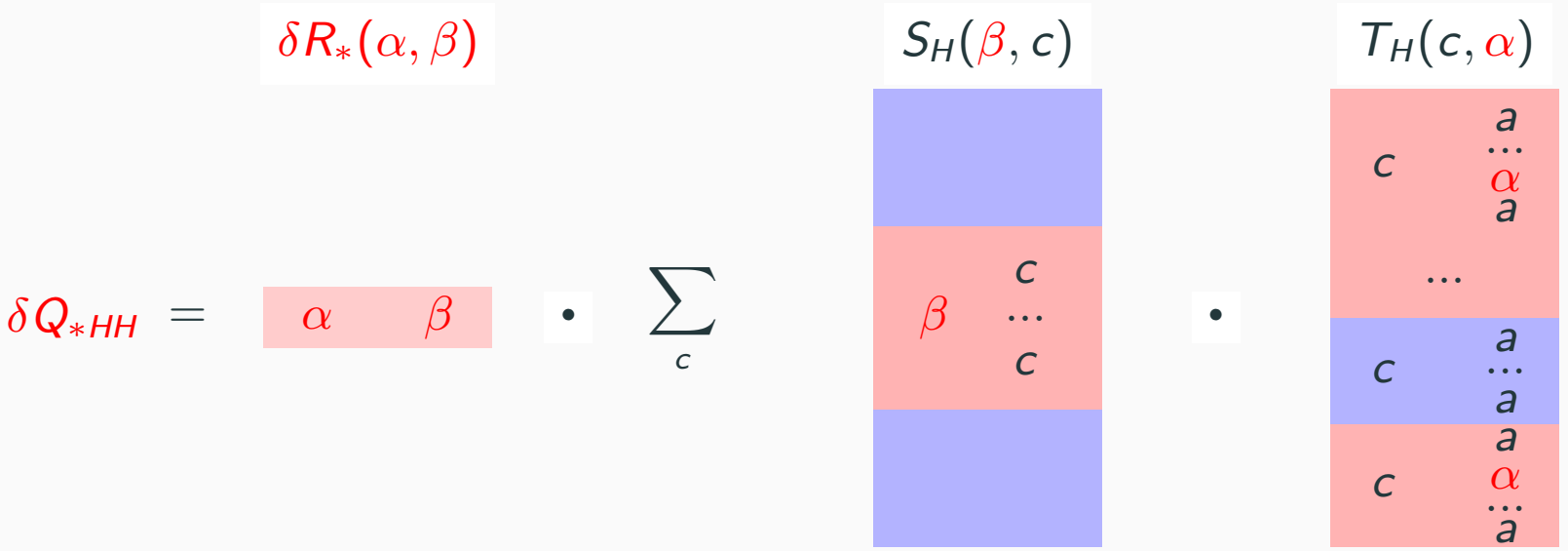


Update time: $\mathcal{O}(N^\varepsilon)$ to intersect the lists of C-values from S_L and T_L

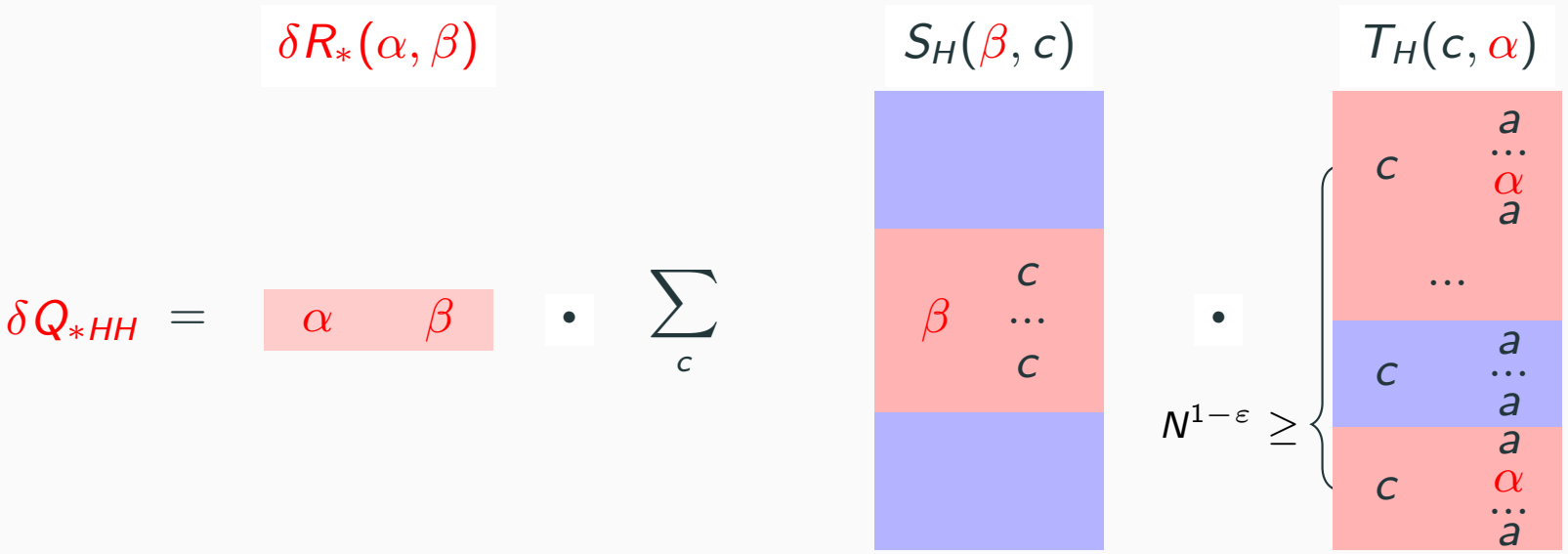
Adaptive Maintenance Strategy

$$\delta Q_{*HH} = \delta R_*(\alpha, \beta) \cdot \sum_c S_H(\beta, c) \cdot T_H(c, \alpha)$$



Adaptive Maintenance Strategy

$$\delta Q_{*HH} = \delta R_*(\alpha, \beta) \cdot \sum_c S_H(\beta, c) \cdot T_H(c, \alpha)$$



Adaptive Maintenance Strategy

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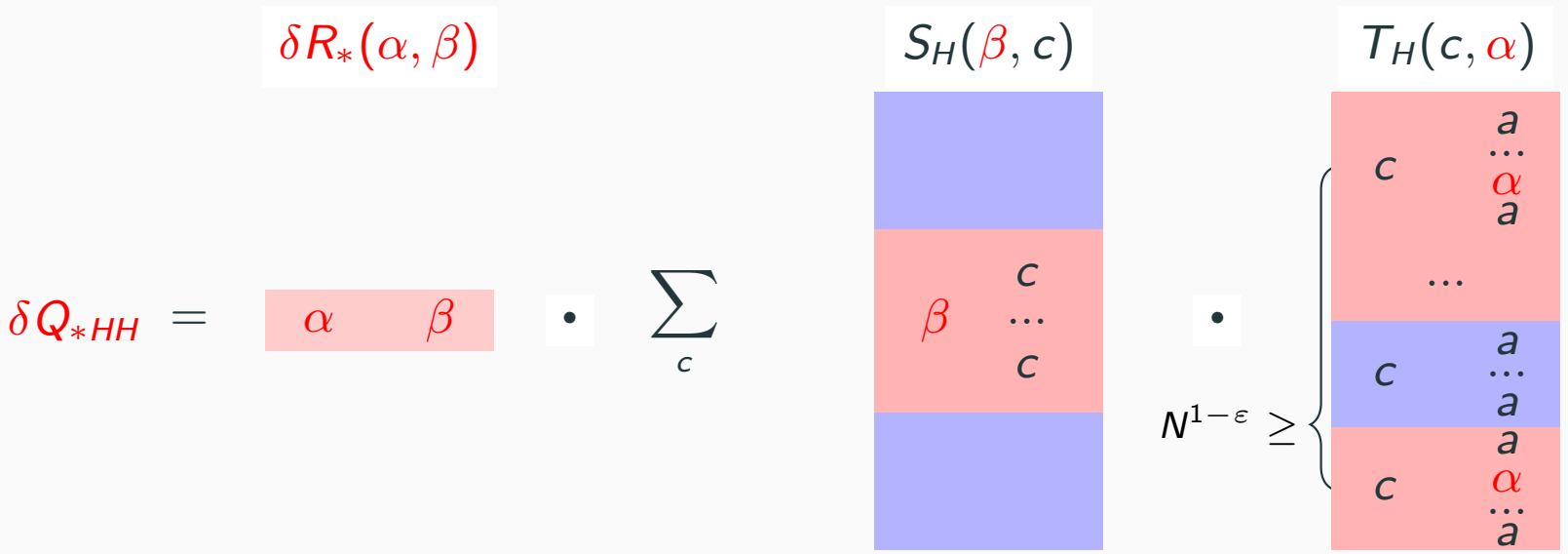
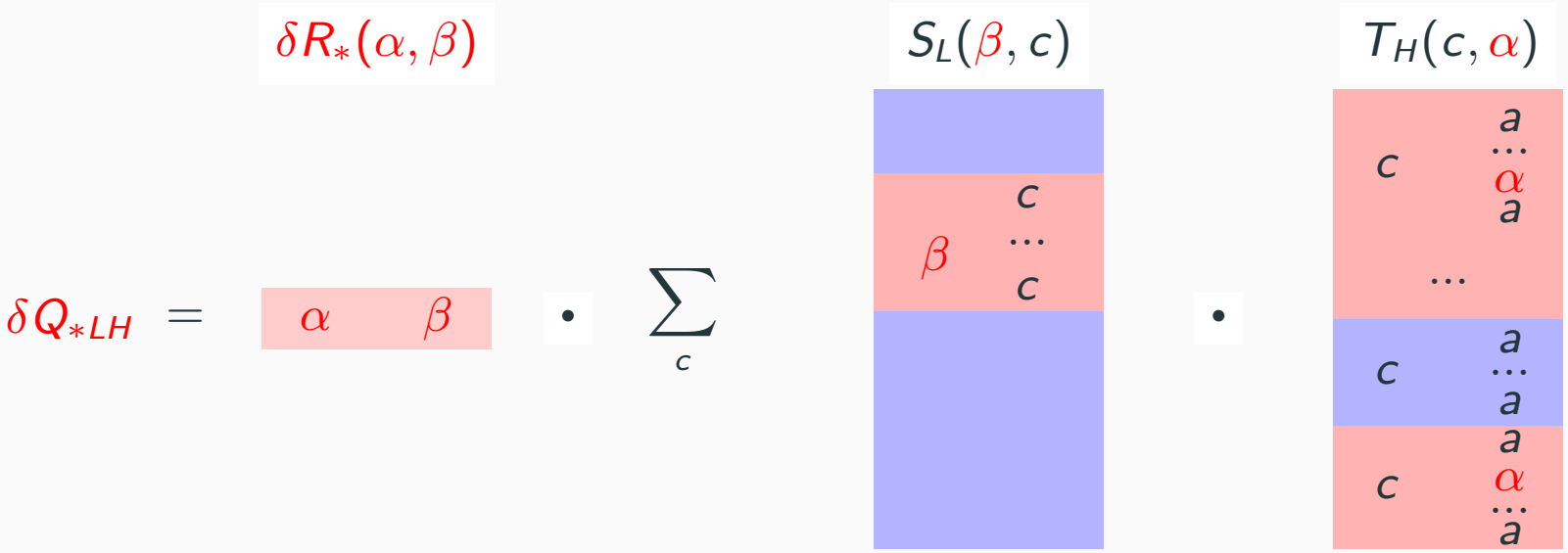


Update time: $\mathcal{O}(N^{1-\varepsilon})$ to intersect the lists of C-values from S_H and T_H

Adaptive Maintenance Strategy

$$\delta Q_{*LH} = \delta R_*(\alpha, \beta) \cdot \sum_c S_L(\beta, c) \cdot T_H(c, \alpha)$$



Adaptive Maintenance Strategy

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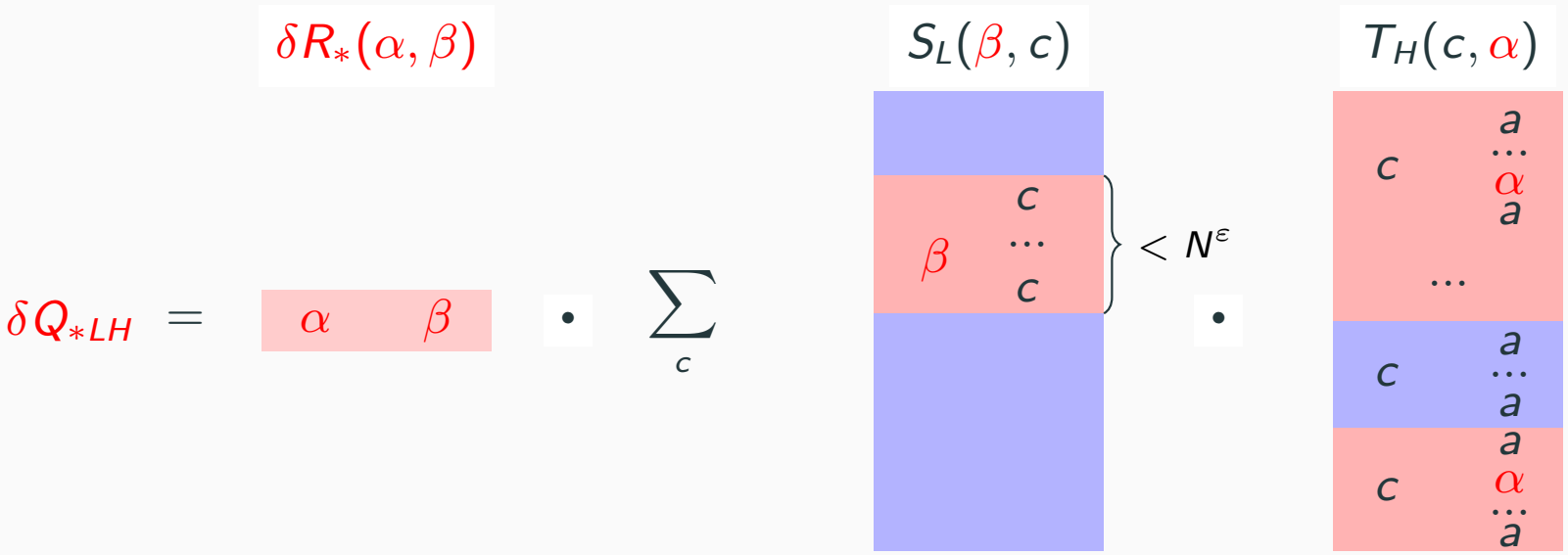
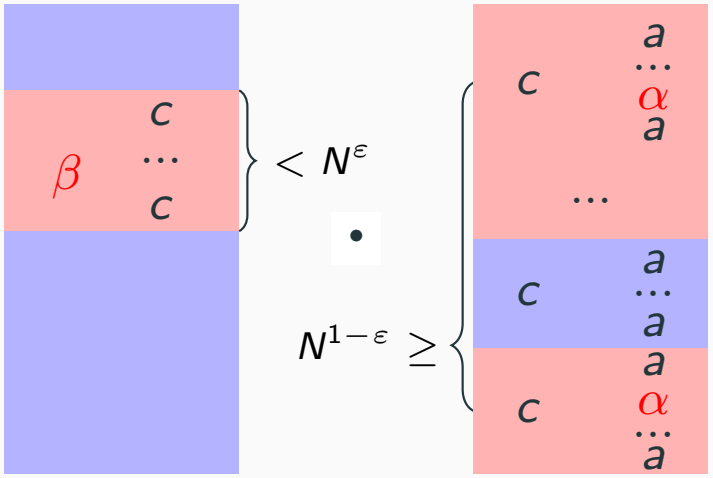


Adaptive Maintenance Strategy

$$\delta Q_{*LH} = \delta R_*(\alpha, \beta) \cdot \sum_c S_L(\beta, c) \cdot T_H(c, \alpha)$$

$\delta R_*(\alpha, \beta)$ $S_L(\beta, c)$ $T_H(c, \alpha)$

$\delta Q_{*LH} =$ α β $\cdot \sum_c$



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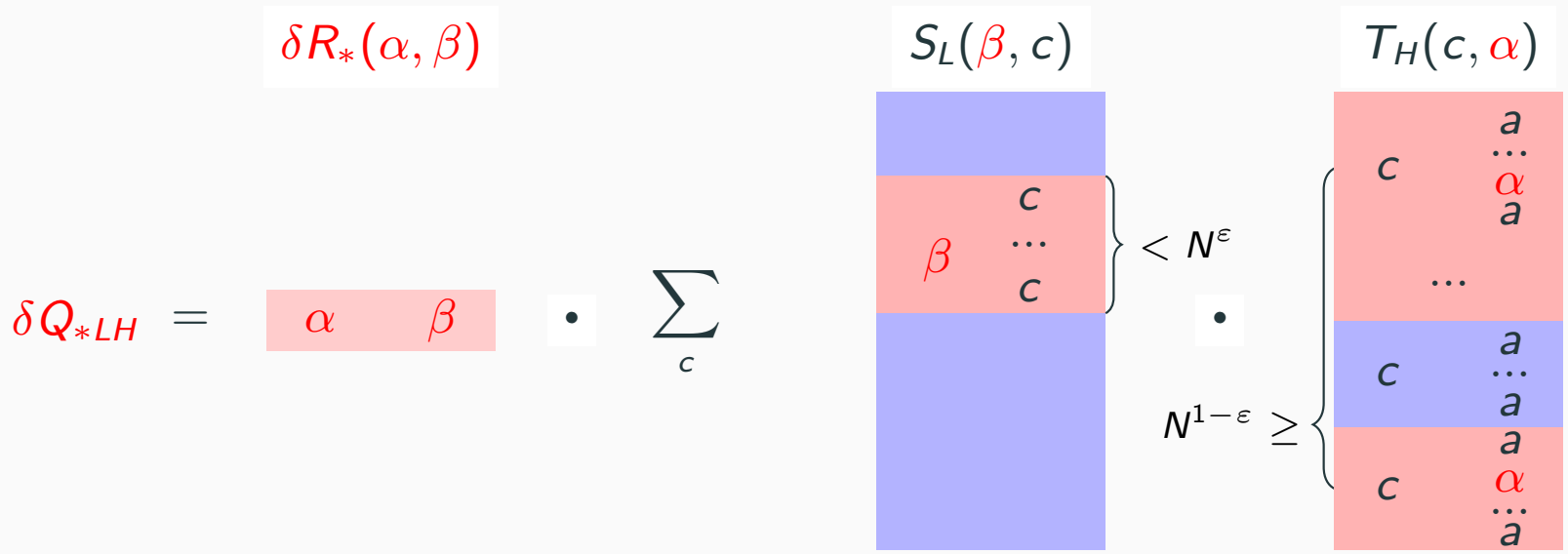

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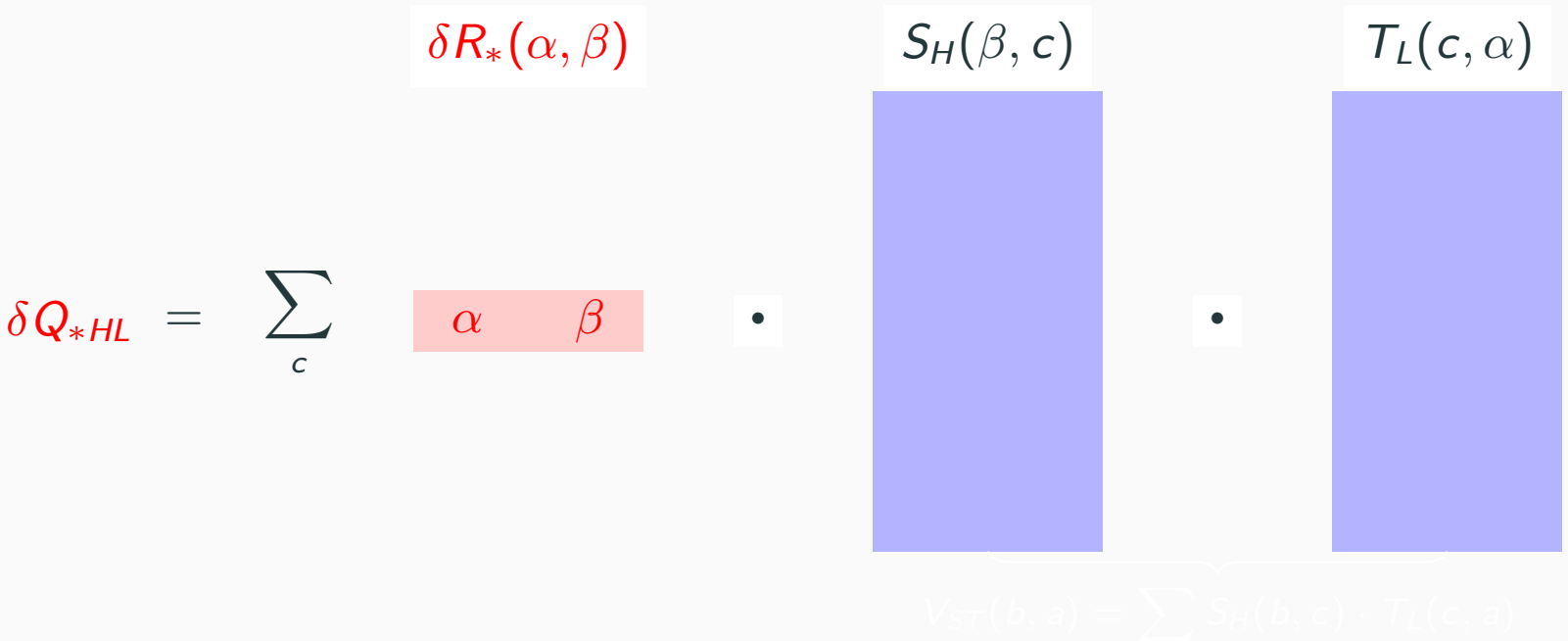

Adaptive Maintenance Strategy

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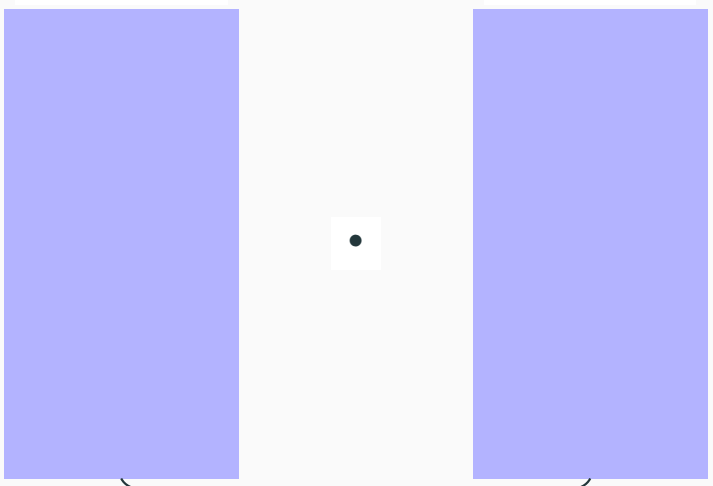
$$\delta Q_{*HL} = \sum_c \quad \delta R_*(\alpha, \beta) \quad \alpha \quad \beta \quad \cdot \quad S_H(\beta, c) \quad \cdot \quad T_L(c, \alpha)$$



$$V_{ST}(b, a) = \sum_c S_H(b, c) \cdot T_L(c, a)$$

Adaptive Maintenance Strategy

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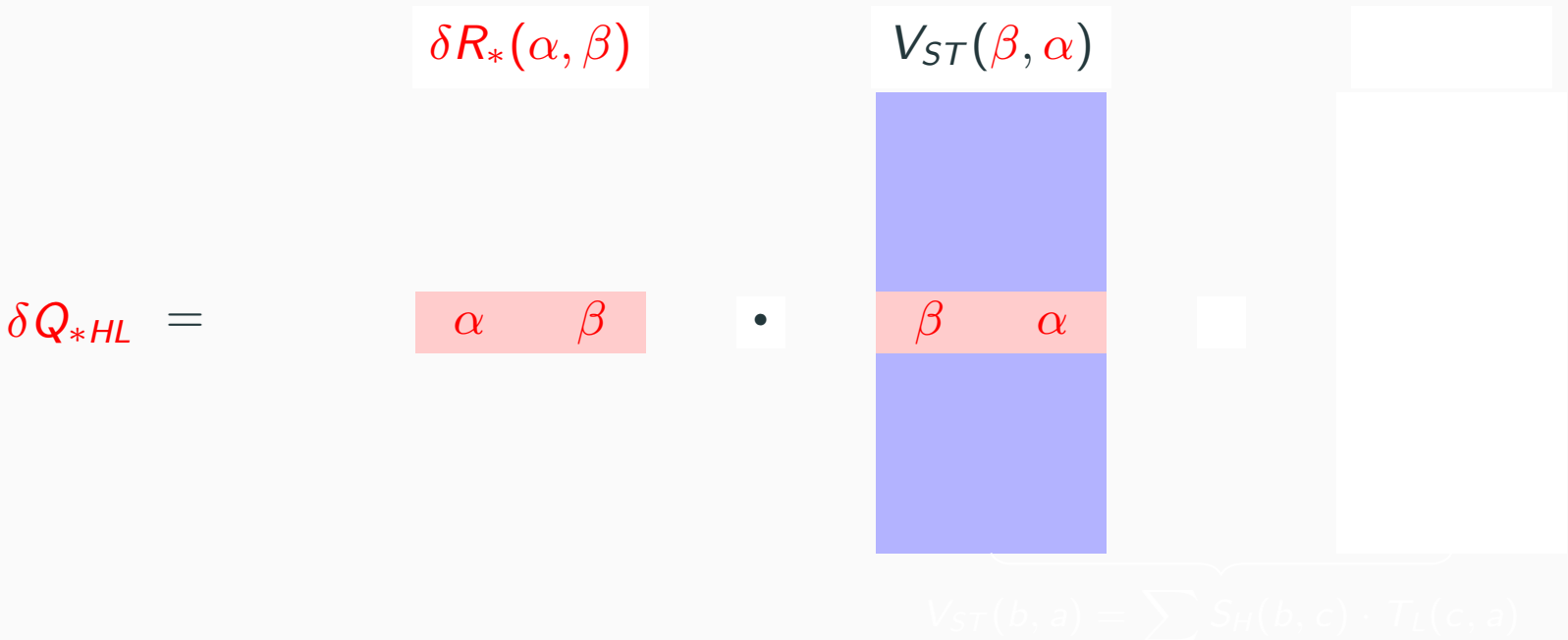

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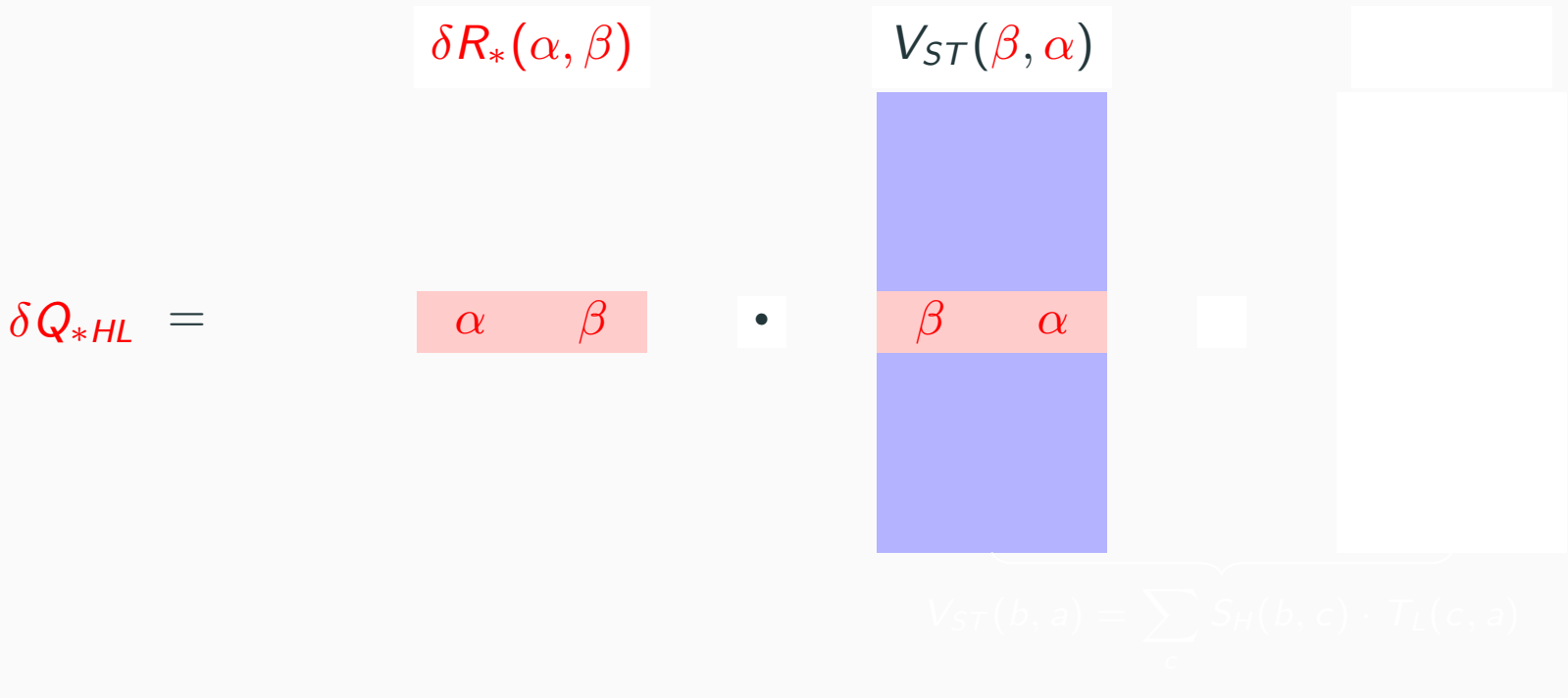


Update time: $\mathcal{O}(1)$ to look up in V_{ST}, assuming V_{ST} is already materialized

Summary of Adaptive Maintenance Strategies

Maintenance for an update $\delta R_* = \{(\alpha, \beta) \mapsto m\}$:

Skew-aware View	Evaluation from left to right	Time
$\sum_{a,b,c} R_*(a,b) \cdot S_L(b,c) \cdot T_L(c,a)$	$\delta R_*(\alpha,\beta) \cdot \sum_c S_L(\beta,c) \cdot T_L(c,\alpha)$	$\mathcal{O}(N^{\varepsilon})$
$\sum_{a,b,c} R_*(a,b) \cdot S_H(b,c) \cdot T_H(c,a)$	$\delta R_*(\alpha,\beta) \cdot \sum_c T_H(c,\alpha) \cdot S_H(\beta,c)$	$\mathcal{O}(N^{1-\varepsilon})$
$\sum_{a,b,c} R_*(a,b) \cdot S_L(b,c) \cdot T_H(c,a)$	$\delta R_*(\alpha,\beta) \cdot \sum_c S_L(\beta,c) \cdot T_H(c,\alpha)$ or $\delta R_*(\alpha,\beta) \cdot \sum_c T_H(c,\alpha) \cdot S_L(\beta,c)$	$\mathcal{O}(N^{\varepsilon})$ $\mathcal{O}(N^{1-\varepsilon})$
$\sum_{a,b,c} R_*(a,b) \cdot S_H(b,c) \cdot T_L(c,a)$	$\delta R_*(\alpha,\beta) \cdot V_{ST}(\beta,\alpha)$	$\mathcal{O}(1)$

Overall update time: $\mathcal{O}(N^{\max(\varepsilon,1-\varepsilon)})$

Auxiliary Materialized Views

$$V_{RS}(a, c) = \sum_{b} R_H(a, b) \cdot S_L(b, c)$$

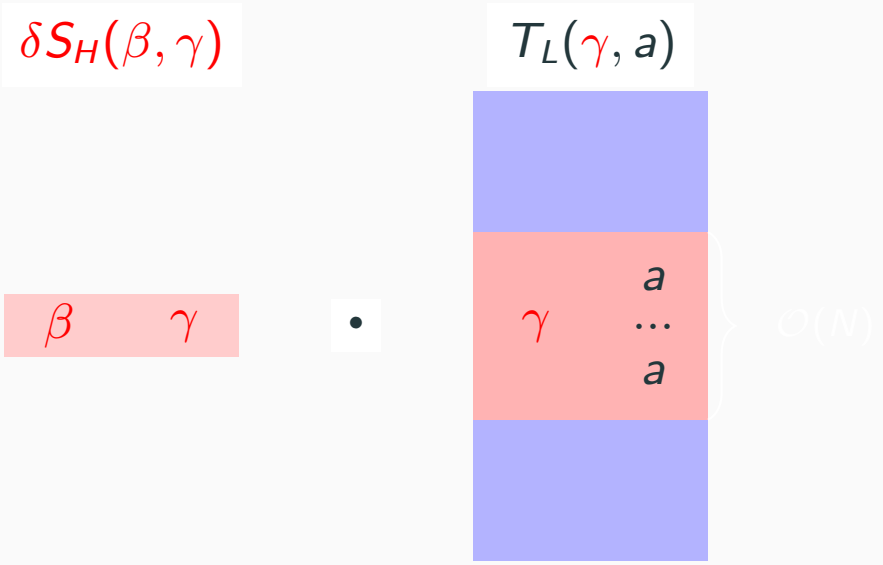
$$V_{ST}(b, a) = \sum_{c} S_H(b, c) \cdot T_L(c, a)$$

$$V_{TR}(a, c) = \sum_{a} T_H(c, a) \cdot R_L(a, b)$$

Maintenance of Auxiliary Views

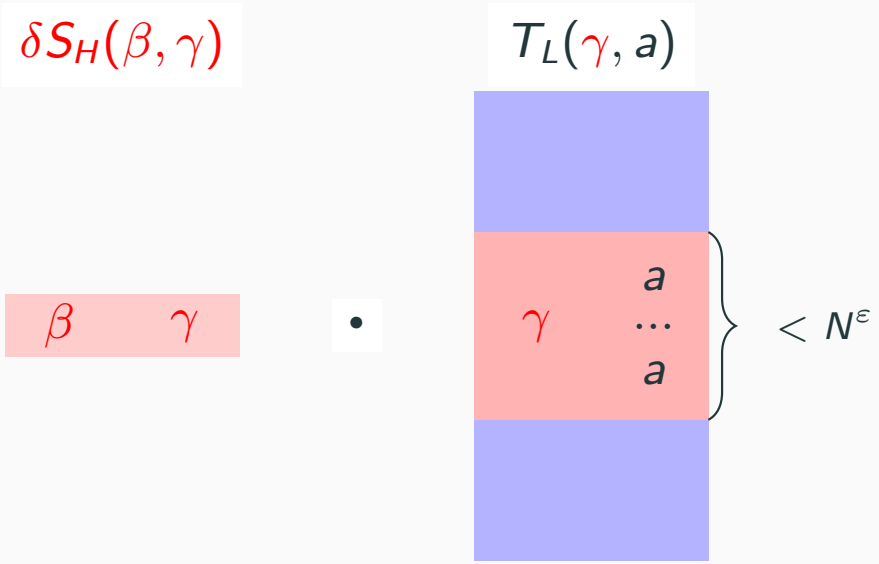
Maintain $V_{ST}(b, a) = \sum_c S_H(b, c) \cdot T_L(c, a)$ under update $\delta S_H = \{(\beta, \gamma) \mapsto m\}$

$\delta V_{ST}(\beta, a) \quad = \quad$



Maintenance of Auxiliary Views

Maintain $V_{ST}(b,a) = \sum_c S_H(b,c) \cdot T_L(c,a)$ under update $\delta S_H = \{(\beta,\gamma) \mapsto m\}$

$\delta V_{ST}(\beta, a) \quad = \quad$ 

Maintenance of Auxiliary Views

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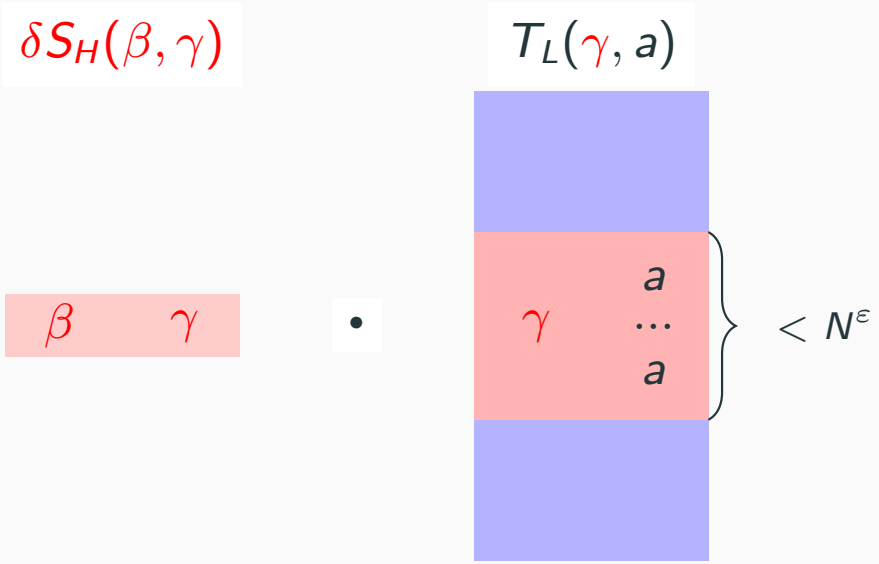
$\delta V_{ST}(\beta, a) \quad = \quad$ 

Update time: $\mathcal{O}(N^{\varepsilon})$ to iterate over a-values paired with γ from T_L

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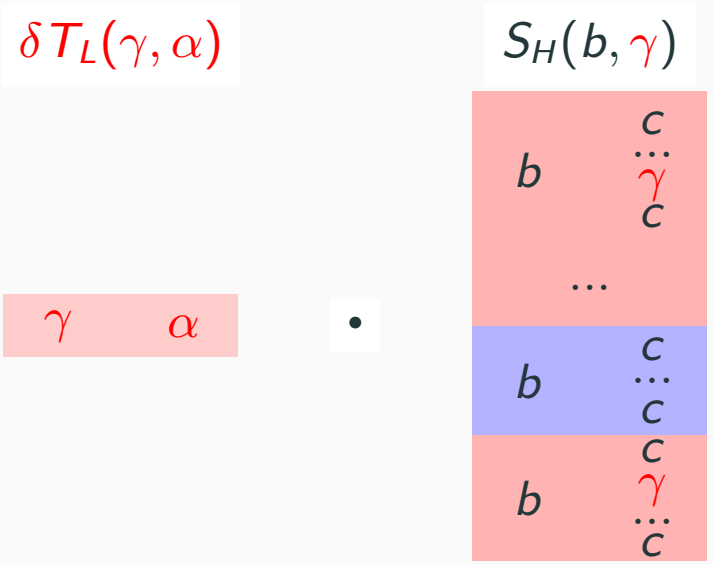
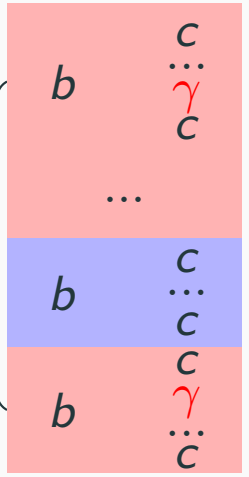
$\delta V_{ST}(b, \alpha) =$ 

Maintenance of Auxiliary Views

Maintain $V_{ST}(b,a) = \sum_c S_H(b,c) \cdot T_L(c,a)$ under update $\delta T_L = \{(\gamma,\alpha) \mapsto m\}$

$\delta V_{ST}(b,\alpha) \quad = \quad \delta T_L(\gamma,\alpha) \quad \gamma \quad \alpha \quad \bullet \quad S_H(b,\gamma)$

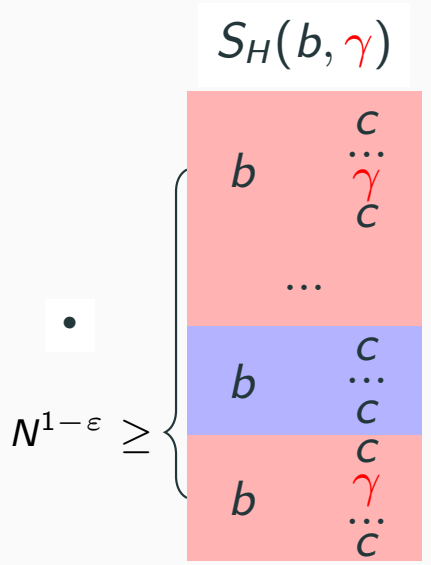
$N^{1-\varepsilon} \geq$



Maintenance of Auxiliary Views

Maintain $V_{ST}(b,a) = \sum_c S_H(b,c) \cdot T_L(c,a)$ under update $\delta T_L = \{(\gamma,\alpha) \mapsto m\}$

$\delta V_{ST}(b,\alpha) \quad = \quad \gamma \quad \alpha \quad \bullet$



Update time: $\mathcal{O}(N^{1-\varepsilon})$ to iterate over b-values paired with γ from S_H

Maintenance of Auxiliary Views: Summary

$$V_{RS}(a,c) = \sum_b R_H(a,b) \cdot S_L(b,c)$$

$$V_{ST}(b,a) = \sum_c S_H(b,c) \cdot T_L(c,a)$$

$$V_{TR}(a,c) = \sum_a T_H(c,a) \cdot R_L(a,b)$$

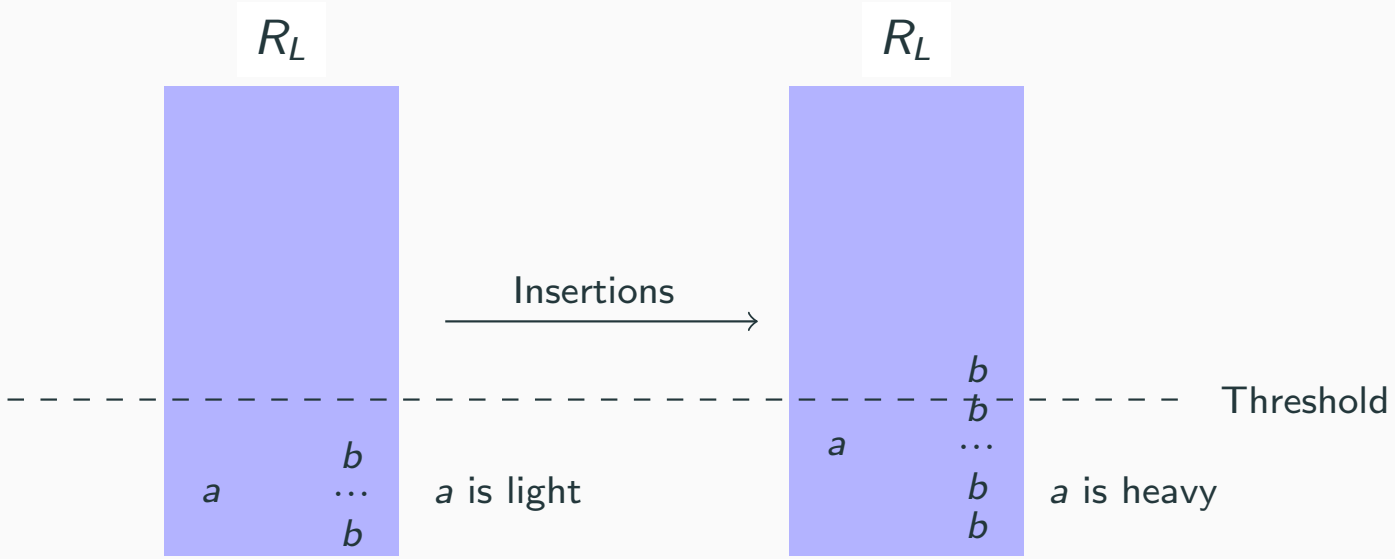
Maintenance Complexity

- Time: $\mathcal{O}(N^{\max\{\varepsilon,1-\varepsilon\}})$
- Space: $\mathcal{O}(N^{1+\min\{\varepsilon,1-\varepsilon\}})$

Updates can change frequencies of values & heavy/light threshold

Rebalancing Partitions

Updates can change the frequencies of values in the factor parts

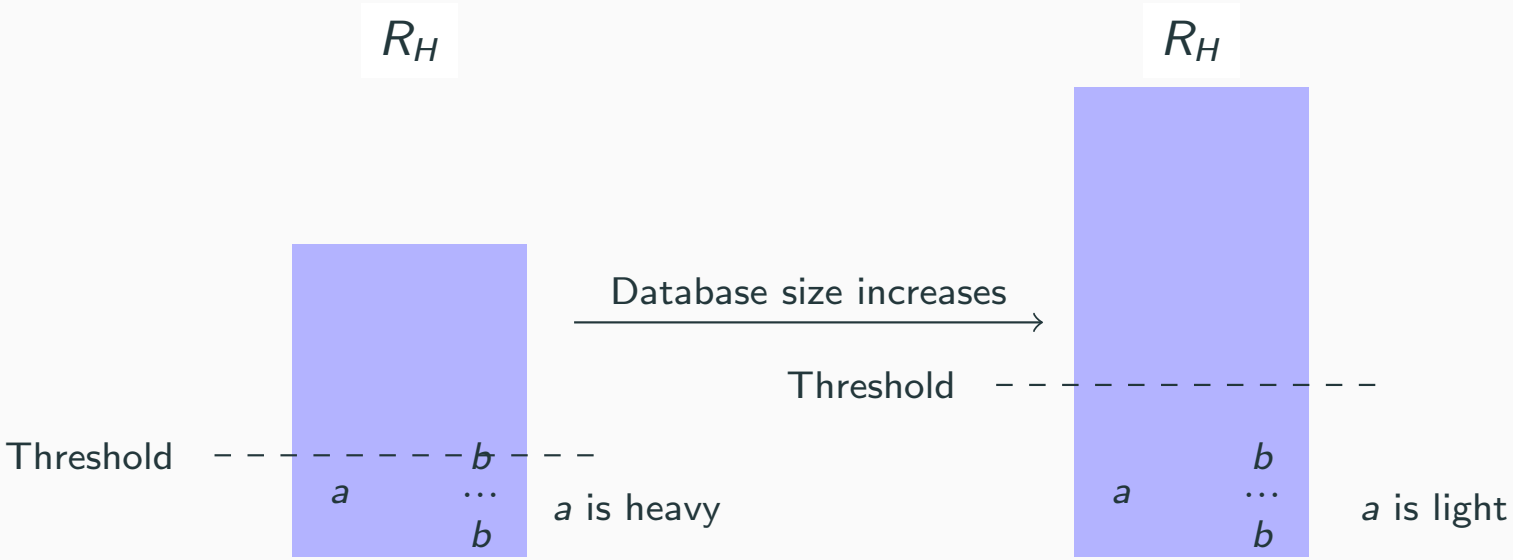


Minor Rebalancing

- Transfer $\mathcal{O}(N^{\varepsilon})$ tuples from one to the other part of the same factor
- Time complexity: $\mathcal{O}(N^{\varepsilon+\max\{\varepsilon,1-\varepsilon\}})$

Rebalancing Partitions

Updates can change the heavy-light threshold!



Major Rebalancing

- Recompute partitions and views from scratch
- Time complexity: $\mathcal{O}(N^{1+\max\{\varepsilon,1-\varepsilon\}})$

Amortization of Rebalancing Times

- Both forms of rebalancing require superlinear time

Amortization of Rebalancing Times

- Both forms of rebalancing require superlinear time
- The rebalancing times amortize over sequences of updates
  - Amortized minor rebalancing time: $\mathcal{O}(N^{\max\{\varepsilon,1-\varepsilon\}})$
  - Amortized major rebalancing time: $\mathcal{O}(N^{\max\{\varepsilon,1-\varepsilon\}})$

$$\ldots \; \textit{update} \quad \overset{\mathcal{O}(N^{\varepsilon+\max\{\varepsilon,1-\varepsilon\}})}{\textit{minor}} \quad \underbrace{\textit{update} \; \ldots \; \textit{update}}_{\Omega(N^{\varepsilon})} \quad \overset{\mathcal{O}(N^{\varepsilon+\max\{\varepsilon,1-\varepsilon\}})}{\textit{minor}} \quad \textit{update} \; \ldots$$

$$\ldots \; \textit{update} \quad \overset{\mathcal{O}(N^{1+\max\{\varepsilon,1-\varepsilon\}})}{\textit{major}} \quad \underbrace{\textit{update} \; \ldots \; \textit{update}}_{\Omega(N)} \quad \overset{\mathcal{O}(N^{1+\max\{\varepsilon,1-\varepsilon\}})}{\textit{major}} \quad \textit{update} \; \ldots$$

Amortization of Rebalancing Times

- Both forms of rebalancing require superlinear time
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  - Amortized minor rebalancing time: $\mathcal{O}(N^{\max\{\varepsilon,1-\varepsilon\}})$
  - Amortized major rebalancing time: $\mathcal{O}(N^{\max\{\varepsilon,1-\varepsilon\}})$
- Overall amortized rebalancing time: $\mathcal{O}(N^{\max\{\varepsilon,1-\varepsilon\}})$

$$\ldots \quad \underset{\mathcal{O}(N^{\varepsilon+\max\{\varepsilon,1-\varepsilon\}})}{\textit{update}} \quad \textit{minor} \quad \underbrace{\textit{update} \quad \ldots \quad \textit{update}}_{\Omega(N^{\varepsilon})} \quad \underset{\mathcal{O}(N^{\varepsilon+\max\{\varepsilon,1-\varepsilon\}})}{\textit{minor}} \quad \textit{update} \quad \ldots$$

$$\ldots \quad \underset{\mathcal{O}(N^{1+\max\{\varepsilon,1-\varepsilon\}})}{\textit{update}} \quad \textit{major} \quad \underbrace{\textit{update} \quad \ldots \quad \textit{update}}_{\Omega(N)} \quad \underset{\mathcal{O}(N^{1+\max\{\varepsilon,1-\varepsilon\}})}{\textit{major}} \quad \textit{update} \quad \ldots$$

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Part 3.

IVM Optimality

IVM Optimality

Two query classes with known worst-case optimal IVM algorithms

- Q-hierarchical queries
  - The only queries that admit constant update time and delay
- δ_1-hierarchical queries
  - They admit $\mathcal{O}(\sqrt{N})$ update time and delay

Query examples shown in Part 2

Part 3.

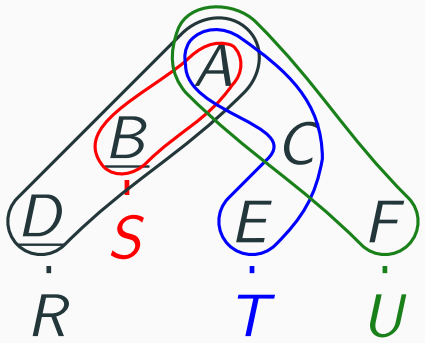
IVM Optimality:

Constant Update Time & Delay

Hierarchical Queries

A query is hierarchical if for any two variables, their sets of atoms in the query are either disjoint or one is contained in the other [VLDB 2004]

$$Q(b,d) = \overset{\text{hierarchical}}{\sum_{a,c,e,f}} R(a,b,d) \cdot S(a,b) \cdot T(a,c,e) \cdot U(a,c,f)$$



Hierarchical Queries

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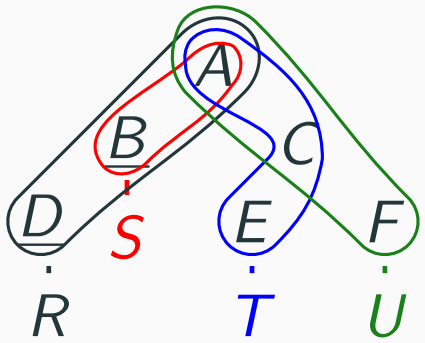
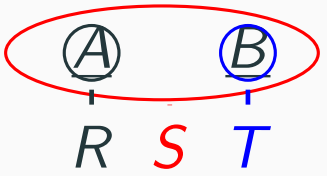
hierarchical

$$Q(b,d) = \sum_{a,c,e,f} R(a,b,d) \cdot S(a,b) \cdot T(a,c,e) \cdot U(a,c,f)$$



not hierarchical

$$Q(a,b) = R(a) \cdot S(a,b) \cdot T(b)$$



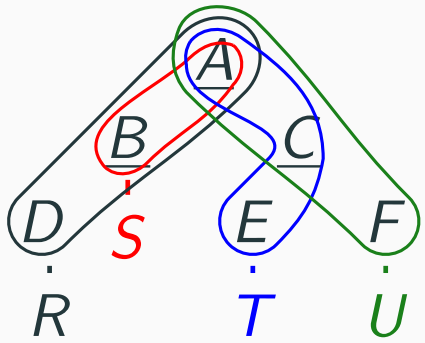
Q-Hierarchical Queries

A query is q-hierarchical if it is hierarchical and the free variables dominate the bound variables

[PODS 2017]

q-hierarchical

$$Q(a,b,c) = \sum_{d,e,f} R(a,b,d) \cdot S(a,b) \cdot T(a,c,e) \cdot U(a,c,f)$$



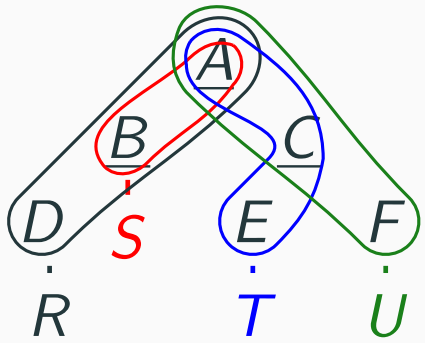
Q-Hierarchical Queries

A query is *q*-hierarchical if it is hierarchical and the free variables dominate the bound variables

[PODS 2017]

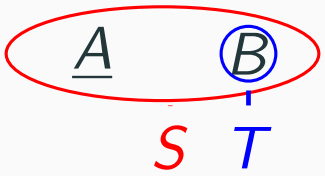
q-hierarchical

$$Q(a,b,c) = \sum_{d,e,f} R(a,b,d) \cdot S(a,b) \cdot T(a,c,e) \cdot U(a,c,f)$$



hierarchical but not *q*-hierarchical

$$Q(a) = \sum_{b} S(a,b) \cdot T(b)$$



Dichotomy for Q-Hierarchical Queries

Let Q be any conjunctive query without self-joins and D a database.

- If Q is **q-hierarchical**, then the query answer admits **O(1)** single-tuple updates and enumeration delay.
- If Q is **not q-hierarchical**, then there is **no algorithm with** $O(|D|^{1/2-\gamma})$ update time and enumeration delay for any $\gamma > 0$, unless the OMv conjecture fails.

[PODS 2017]

Queries under Functional Dependencies

Rewriting queries under functional dependencies [ICDE 2009]

- Given: Query Q and set Σ of functional dependencies
- Replace the set of variables of each atom in Q by its closure under Σ called Σ-reduct

  Under $\Sigma = \{x \to y, y \to z\}$, the closure of $\{x\}$ is $\{x, y, z\}$
- If the Σ-reduct is q-hierarchical, then Q admits constant update time and enumeration delay [VLDB J 2023]

Maintenance of Q-Hierarchical Queries

How to achieve constant update time and enumeration delay?
Recipe: [PODS 2017]

- Construct a factorized representation of the query answer [ICDT 2012]
- Such factorizations admit constant-delay enumeration
- Apply updates directly on the factorization

F-IVM system [`https://github.com/fdbresearch/FIVM`] [SIGMOD 2018]

- Factorize the query answer as a tree of views
- Materialize the views to speed up updates and enumeration

Example: Query Rewriting

$$Q(w, x, y, z) = R(w, x) \cdot S(x, y) \cdot T(y, z)$$

Assume the functional dependencies: $X \to Y$ and $Y \to Z$

Q is not q-hierarchical, but its rewriting under FDs is:

$$Q'(w, x, y, z) = R'(w, x, y, z) \cdot S'(x, y, z) \cdot T'(y, z)$$

Example: Variable Order

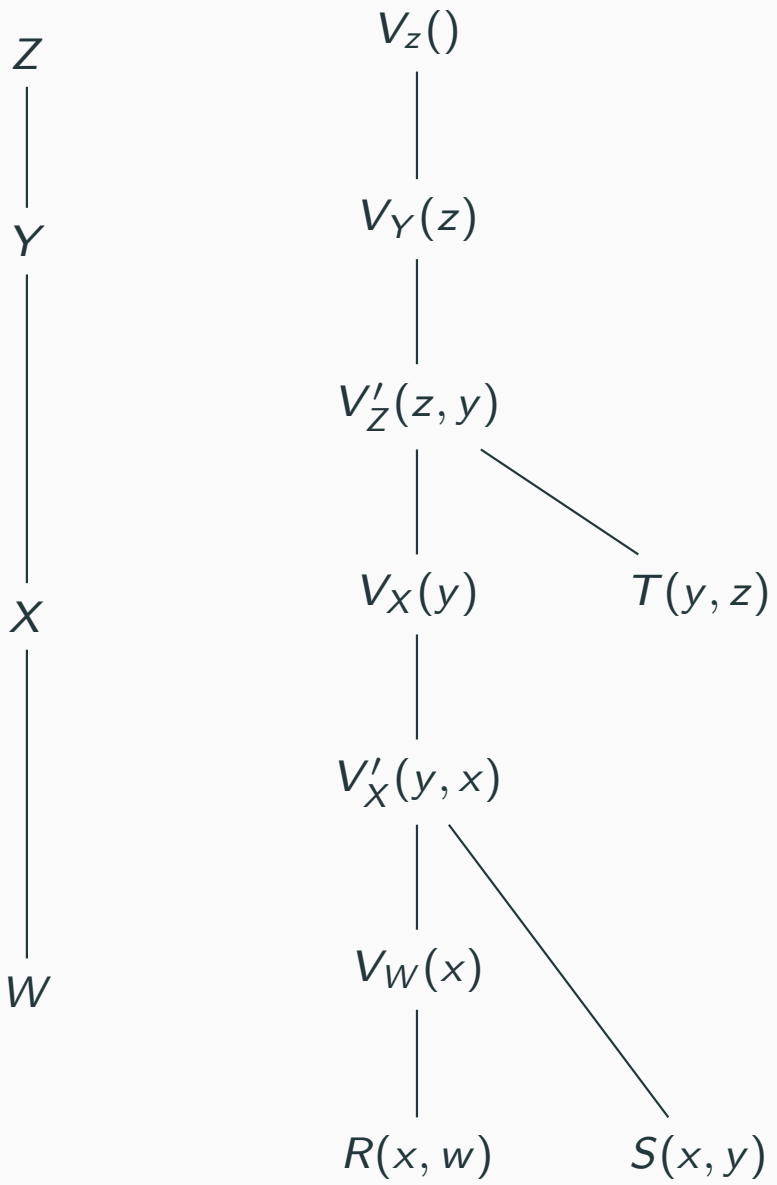
$$Q'(w,x,y,z) = R'(w,x,y,z) \cdot S'(x,y,z) \cdot T'(y,z)$$

Top-down construction of variable order for Q':

- Z and Y are first as they dominate X and W
- Then X, which dominates W
- Finally W

We use this variable order also for Q

Example: View Tree



View tree construction:

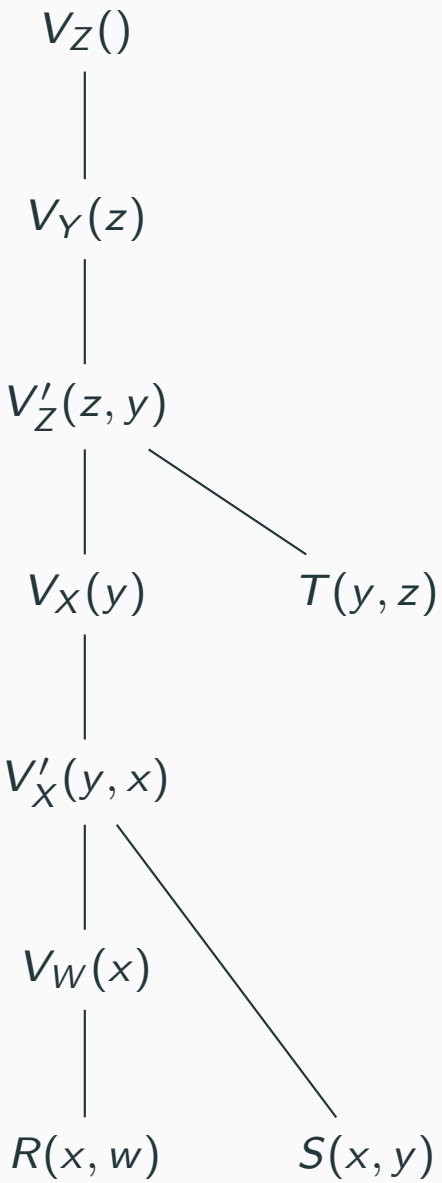
- Place factors at leaves
- Create parent view to join children

$$V'_Z(z,y) = T(y,z) \cdot V_X(y)$$
$$V'_X(y,x) = S(x,y) \cdot V_W(x)$$

- Aggregate away variables not needed for further joins

$$V_Z() = \sum_z V_Y(z)$$
$$V_Y(z) = \sum_y V'_Z(y,z)$$
$$V_X(y) = \sum_x V'_X(x,y)$$
$$V_W(x) = \sum_w R'(x,w)$$

Example: Single-Tuple Update to T



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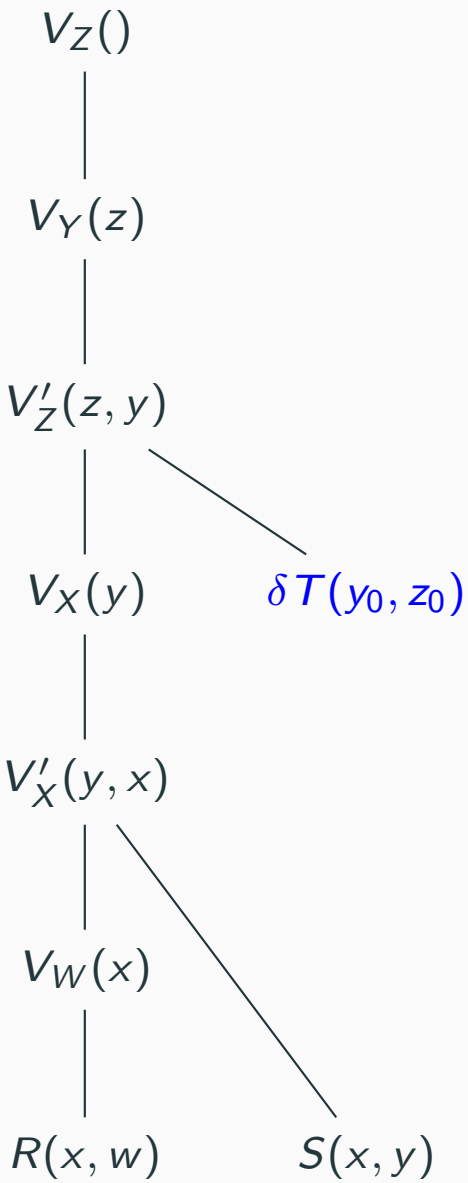

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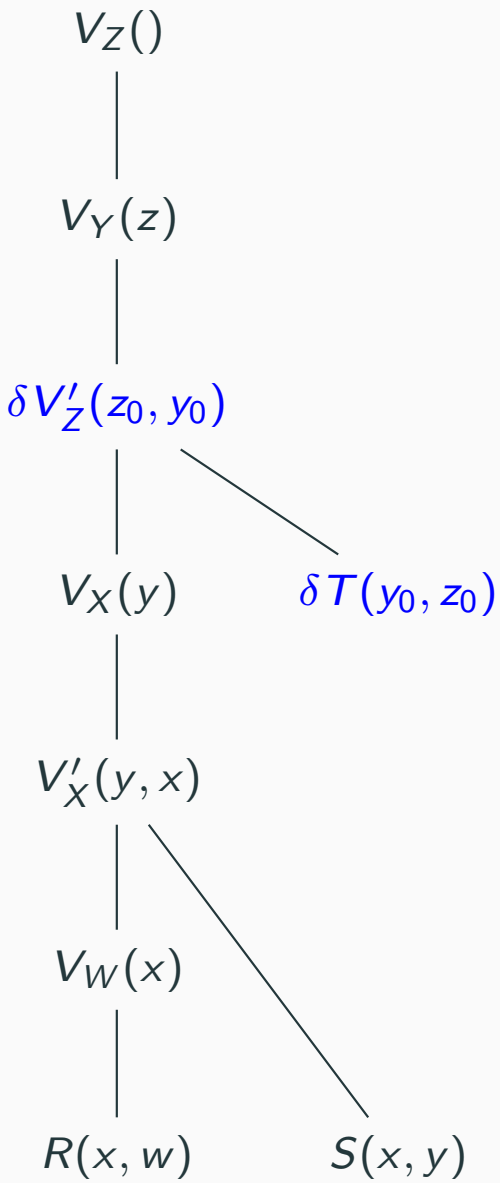

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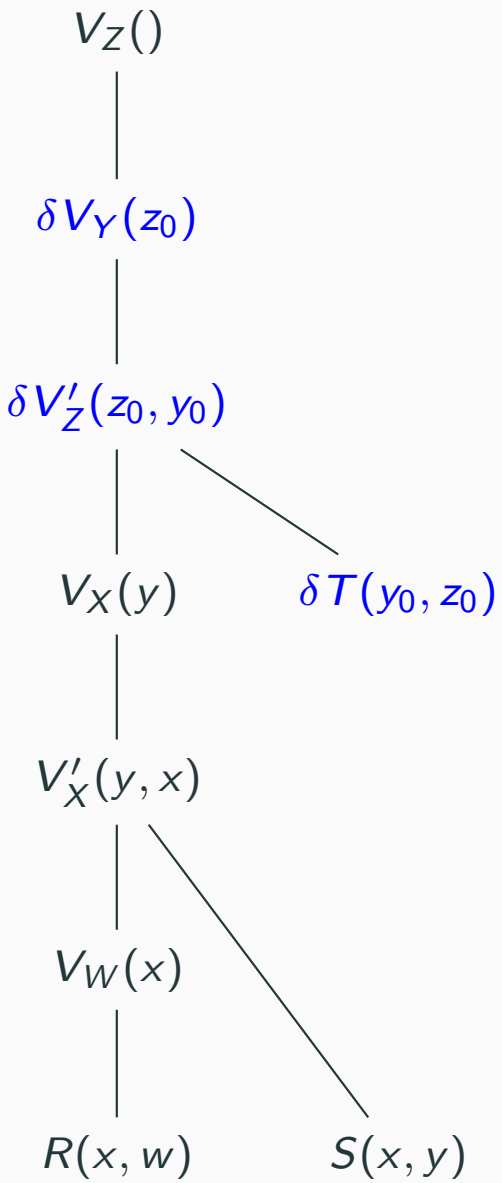

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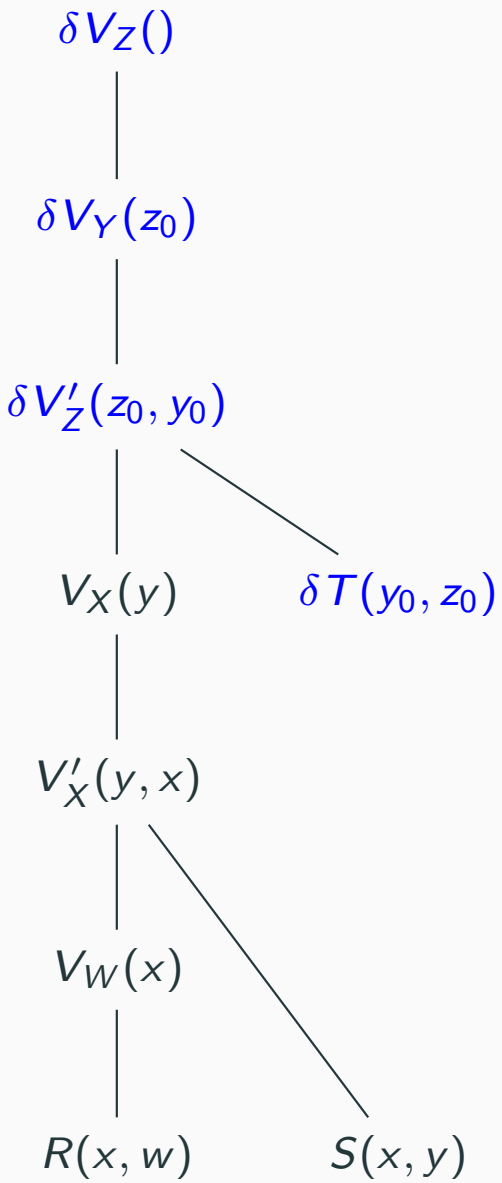

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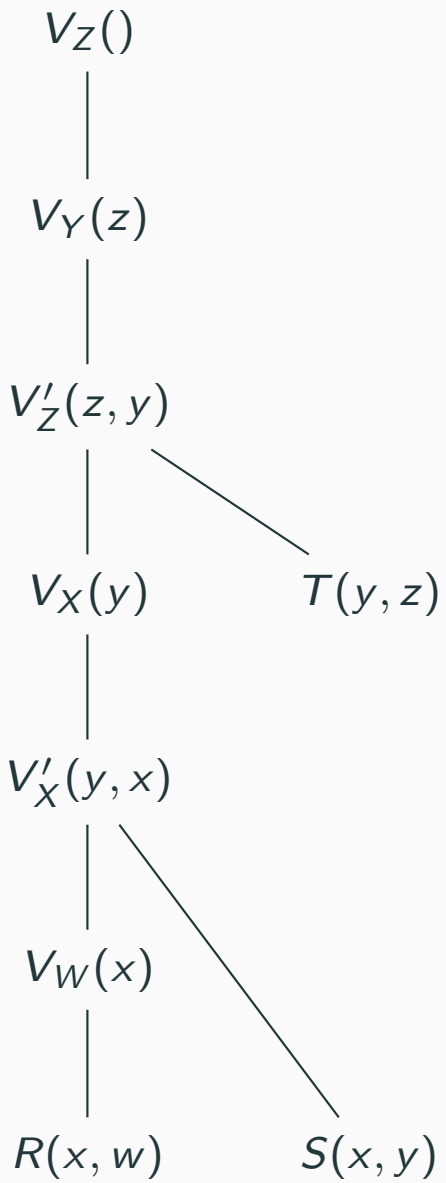

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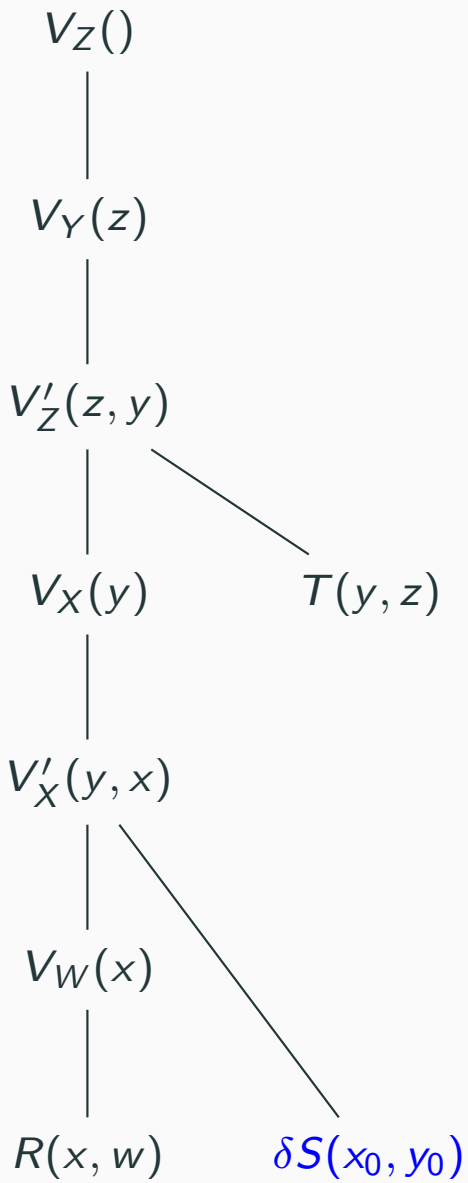

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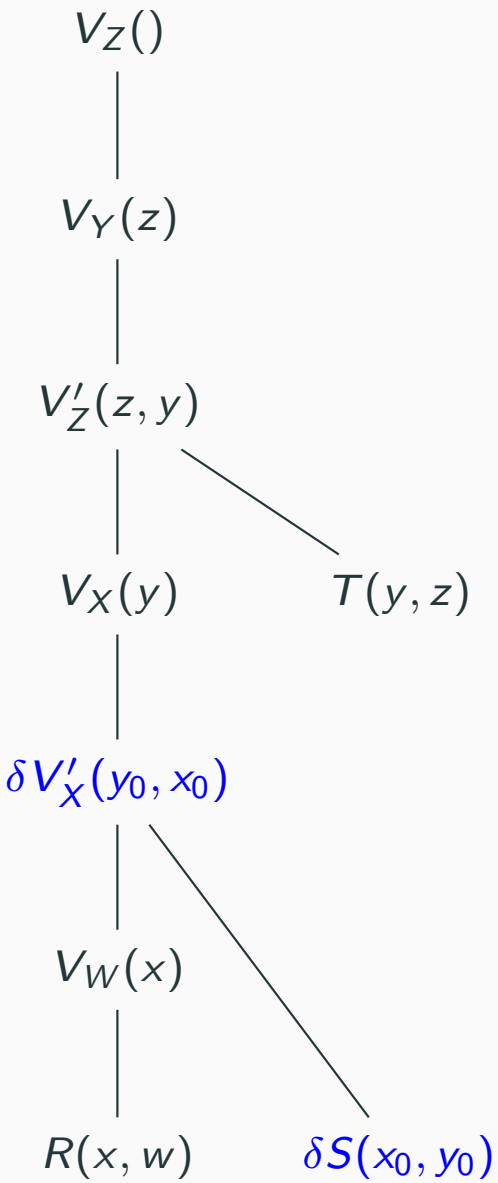

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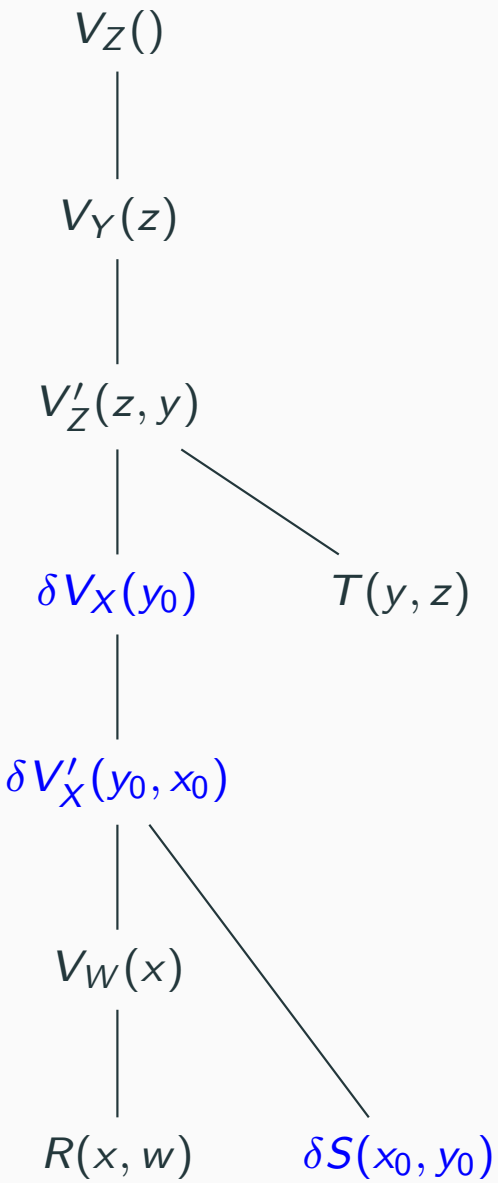

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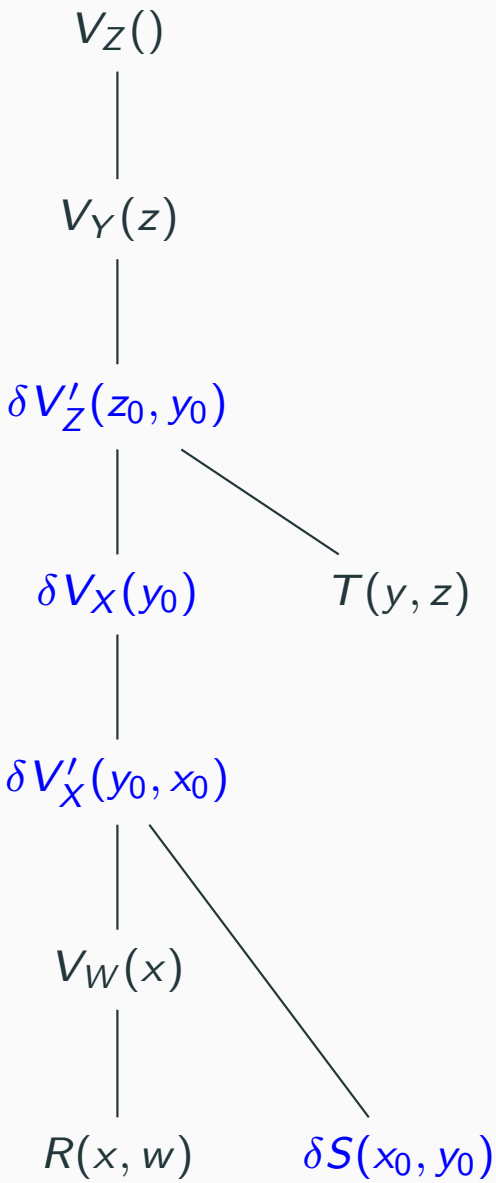

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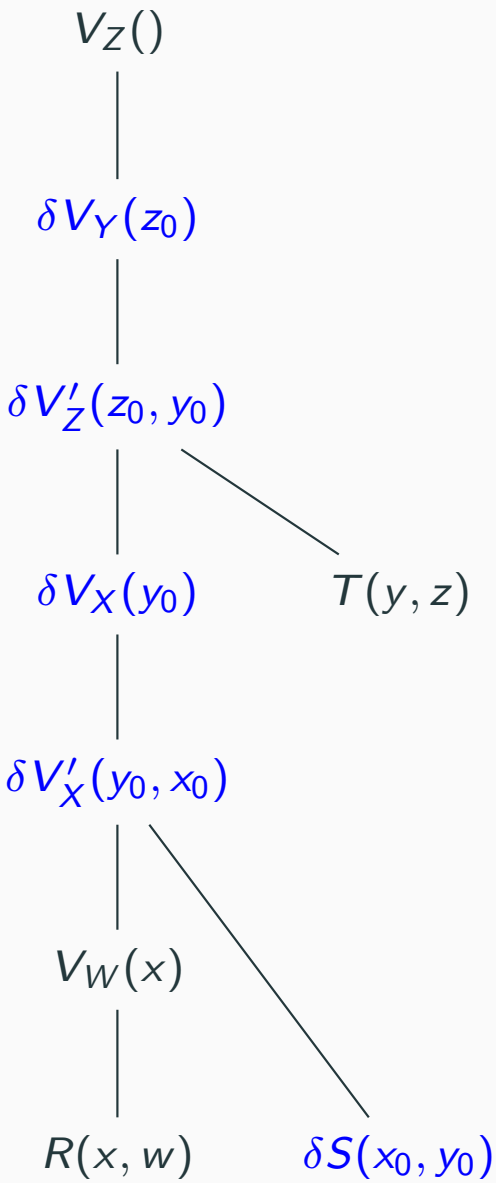

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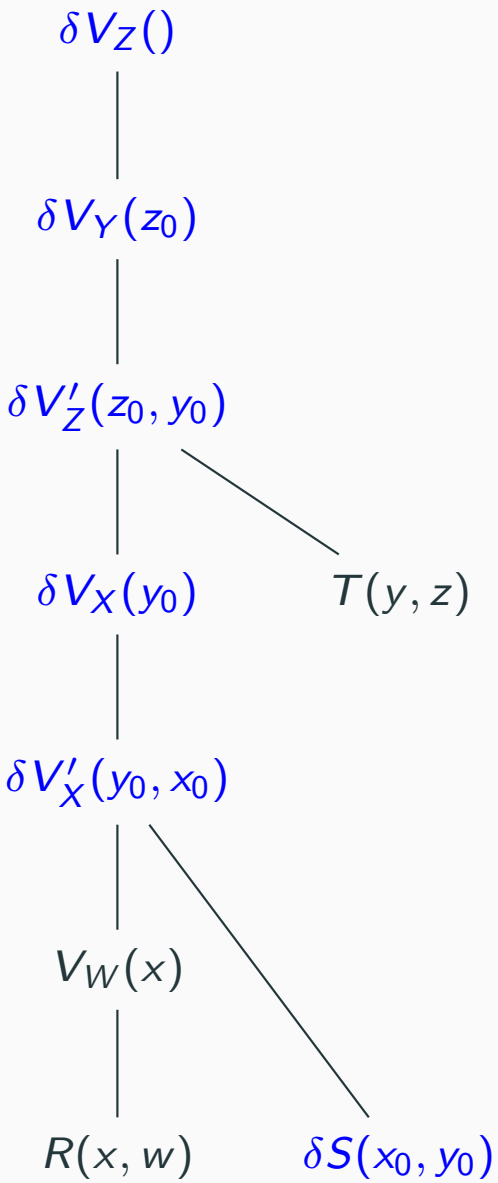

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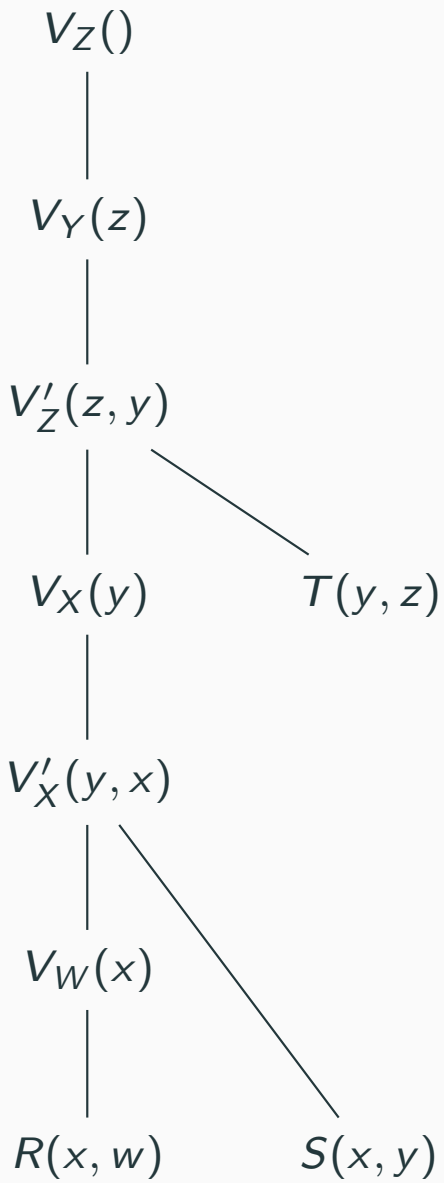

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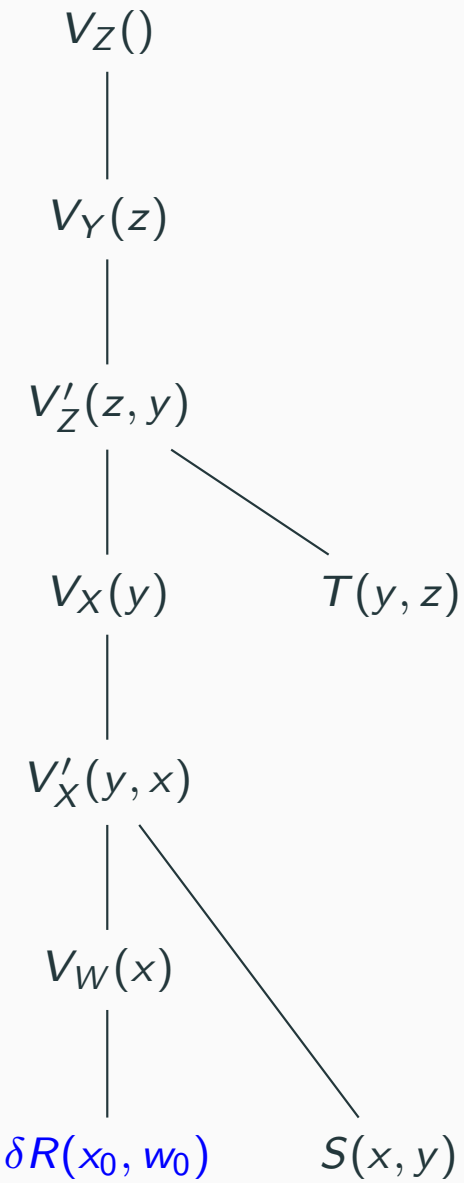

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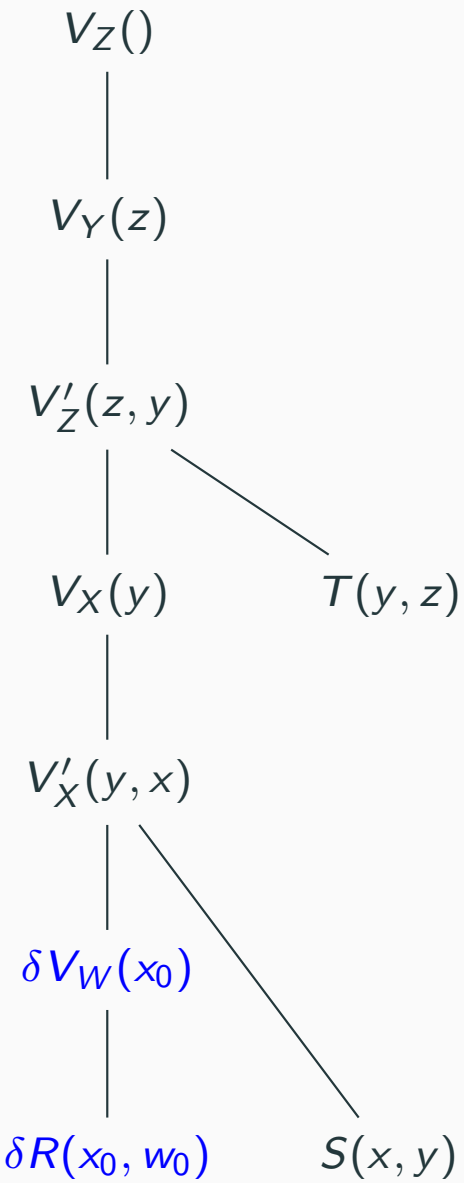

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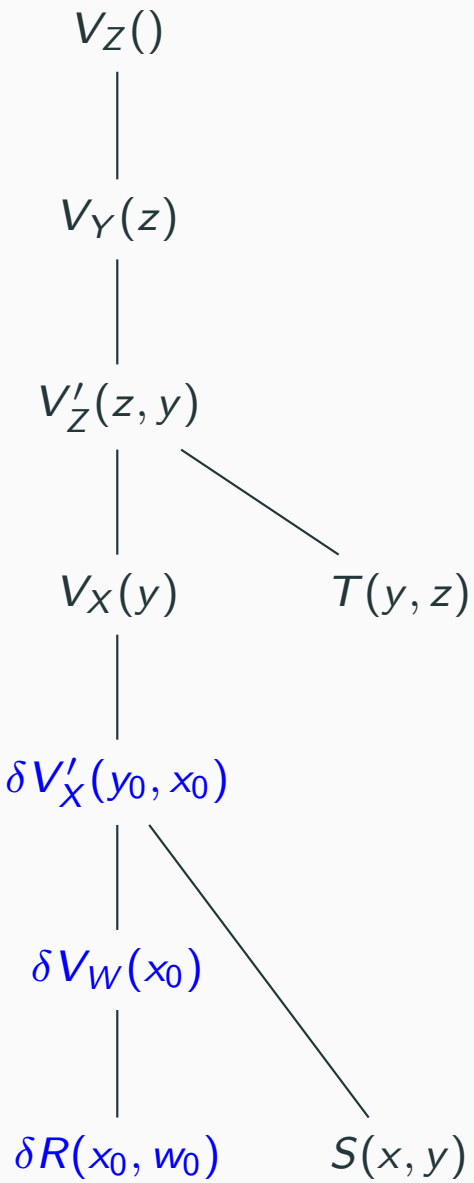

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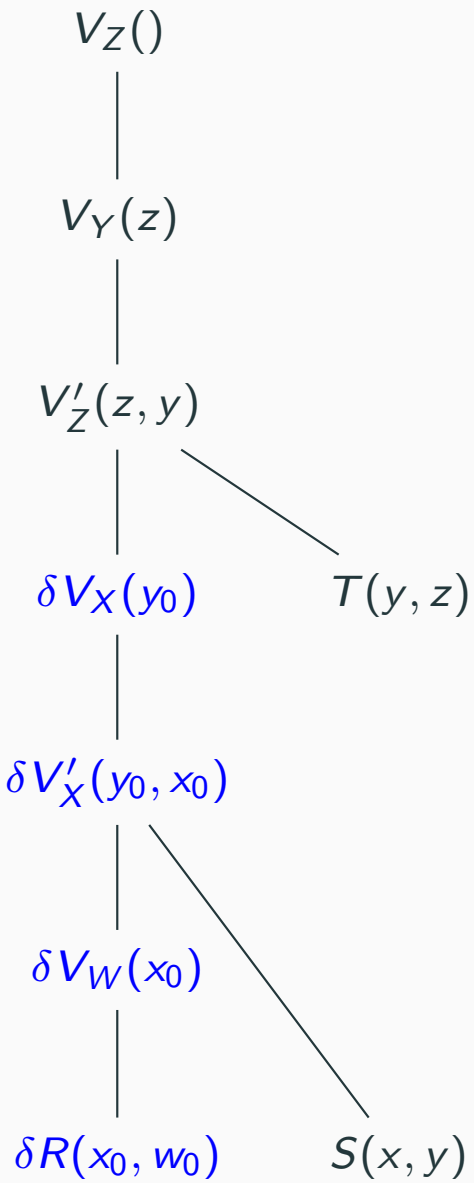

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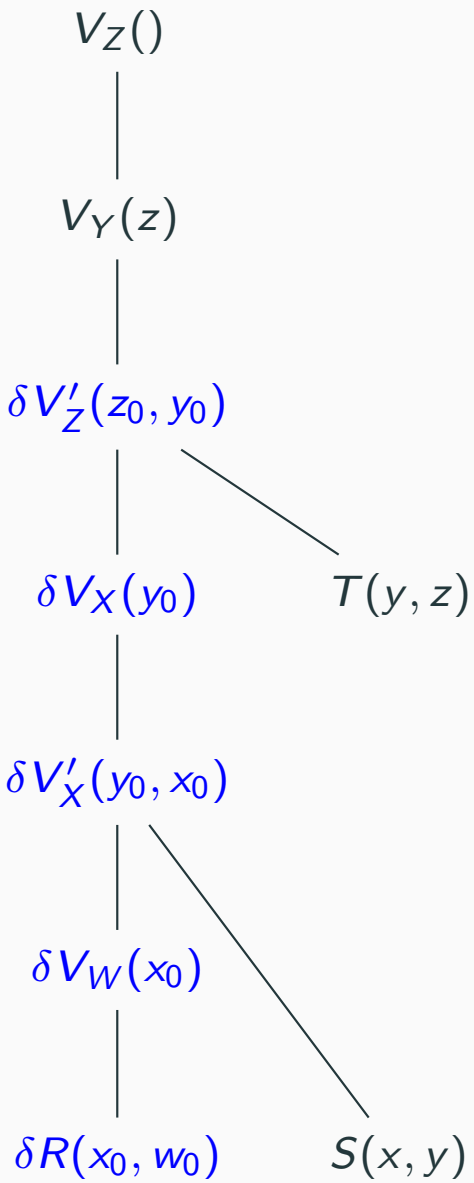

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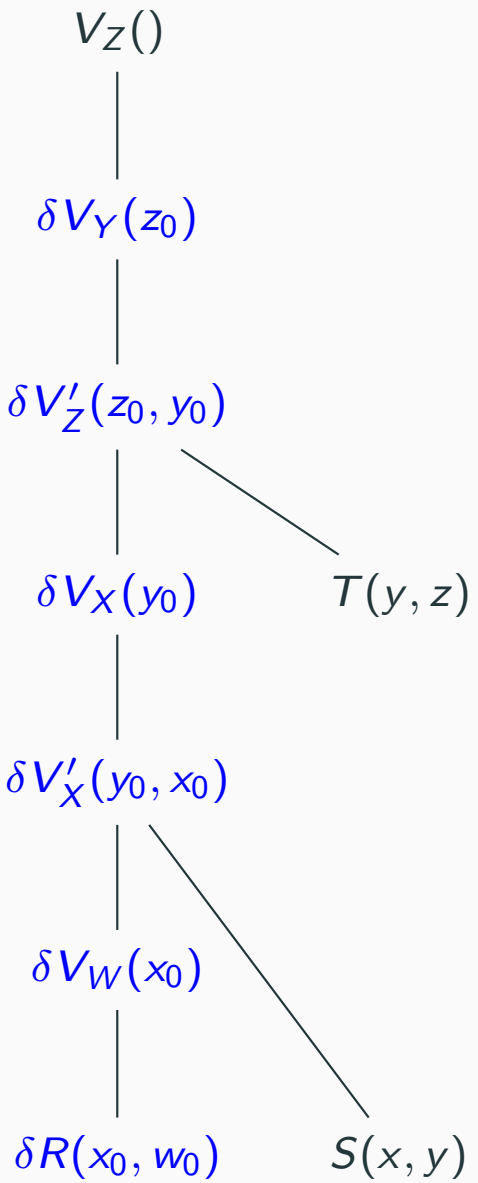

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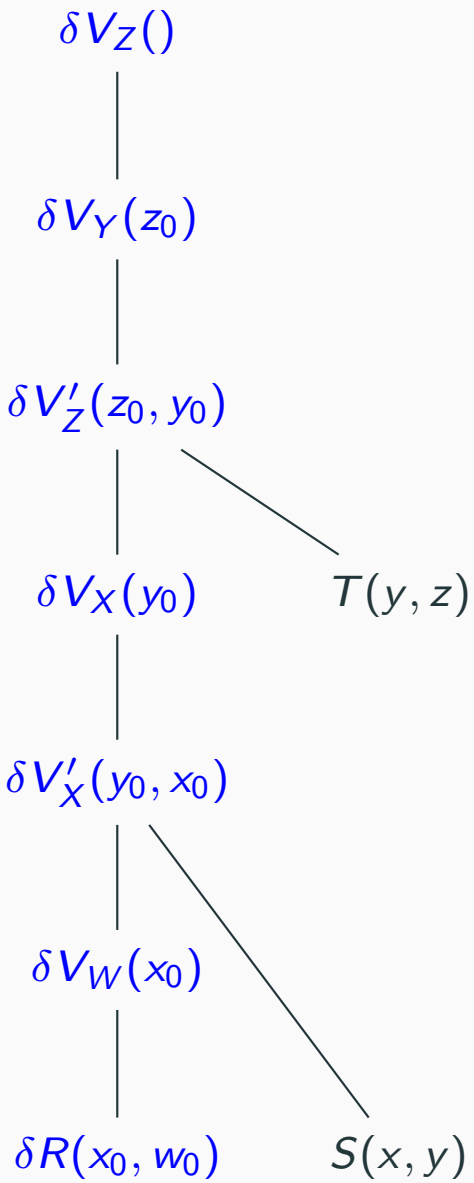

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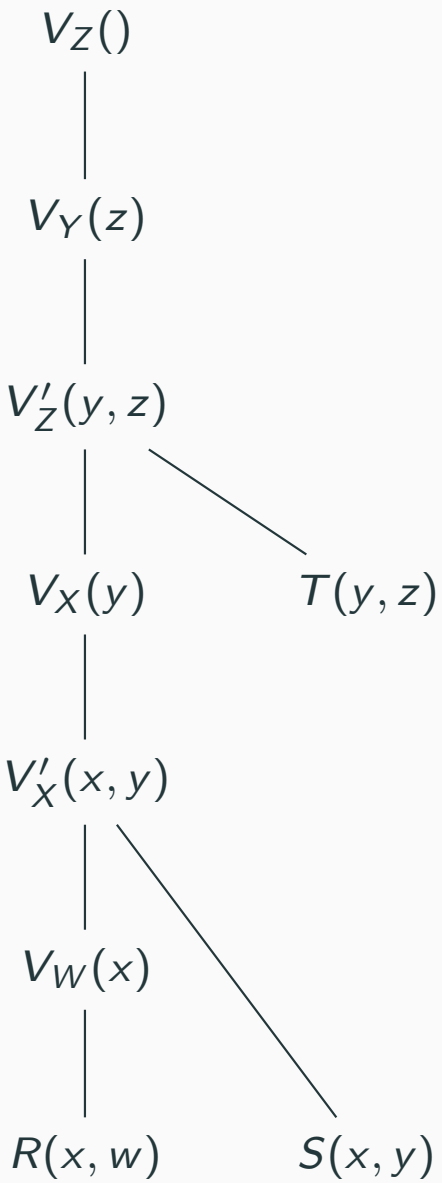

Enumeration for $Q(z,y,x,w)$ with constant delay

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- Views calibrated for variables underneath
- Guaranteed to get matching tuples in views below

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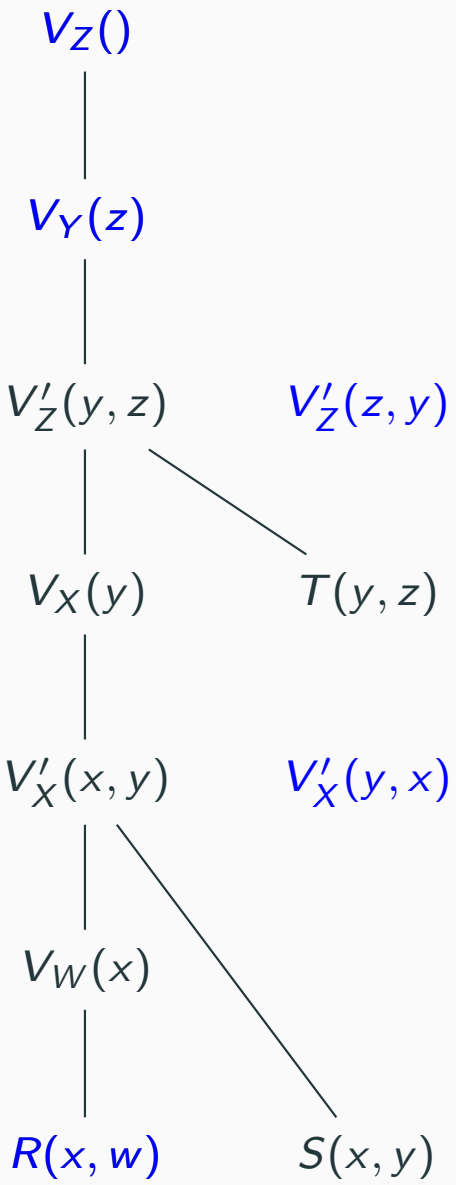

Enumeration for $Q(z,y,x,w)$ with constant delay

- Top-down in the view tree
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The enumeration procedure:

- Is $V_Z()$ empty? If yes, stop.
- Iterate over z's in $V_Y(z)$
- For each z, iterate over y's in index $V'_Z(z,y)$
- For each y, iterate over x's in index $V'_X(y,x)$
- Iterate over $T(z,y)$, $S(x,y)$, $R(x,w)$

Part 3.

IVM Optimality:

$\mathcal{O}(\sqrt{N})$ Update Time & Delay

Simplest Hierarchical Query without "Q" Property

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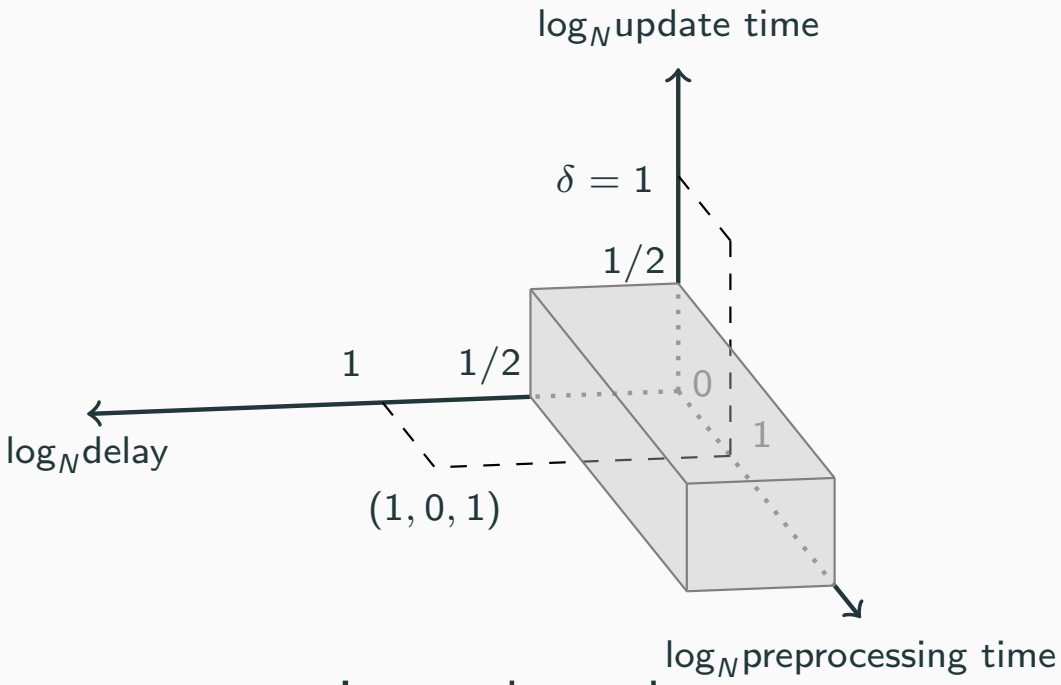

Lower bound

For this query, there is no algorithm that admits

preprocessing time	update time	enumeration delay
arbitrary	$\mathcal{O}(N^{1/2-\gamma})$	$\mathcal{O}(N^{1/2-\gamma})$

for any $\gamma > 0$, unless the OMv Conjecture fails [PODS 2017]

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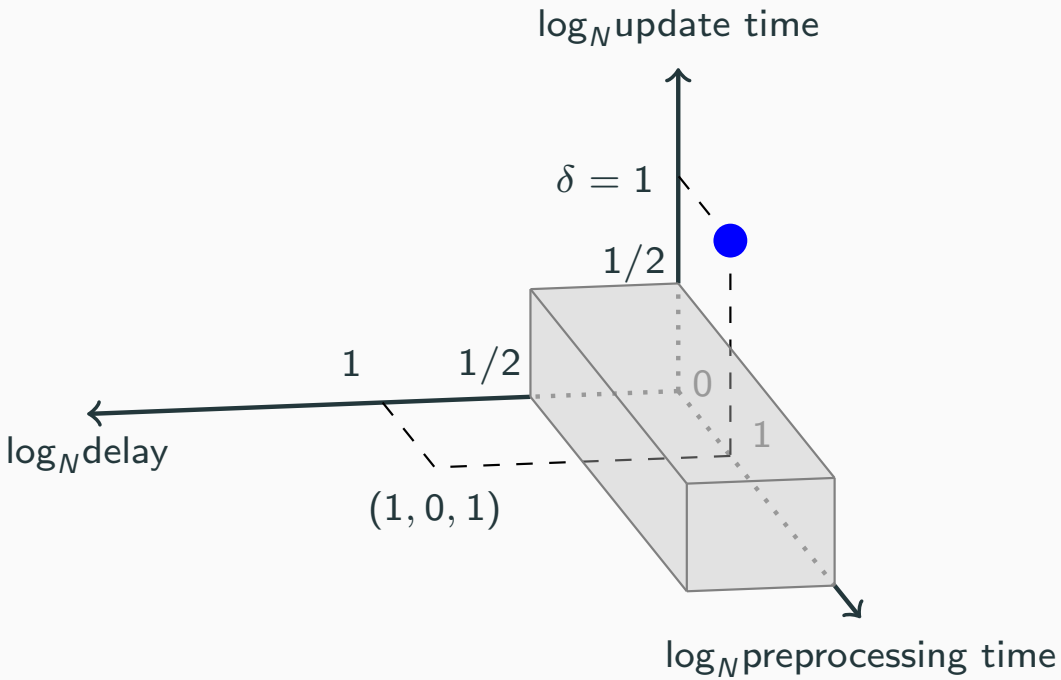

Known approach: Eager update, quick enumeration

- Preprocessing: Materialize the result.
- Upon update: Maintain the materialized result.
- Enumeration: Enumerate from materialized result.

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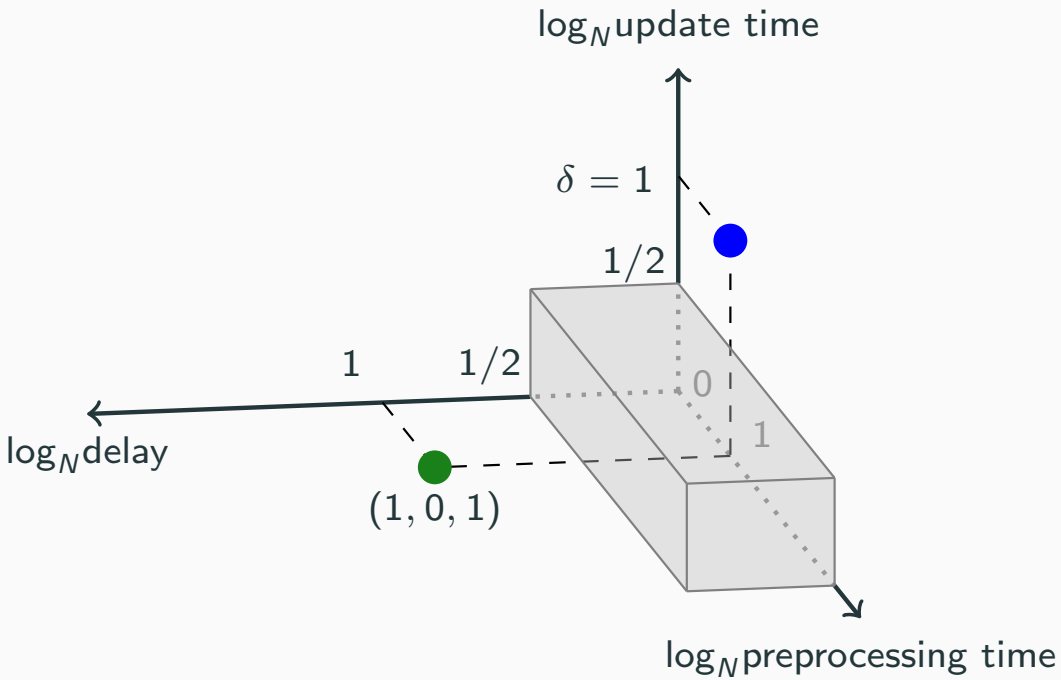

Known approach: Lazy update, heavy enumeration

- Preprocessing: Eliminate dangling tuples
- Upon update: Update only input factors
- Enumeration: Eliminate dangling tuples and enumerate from R

Simplest Hierarchical Query without "Q" Property

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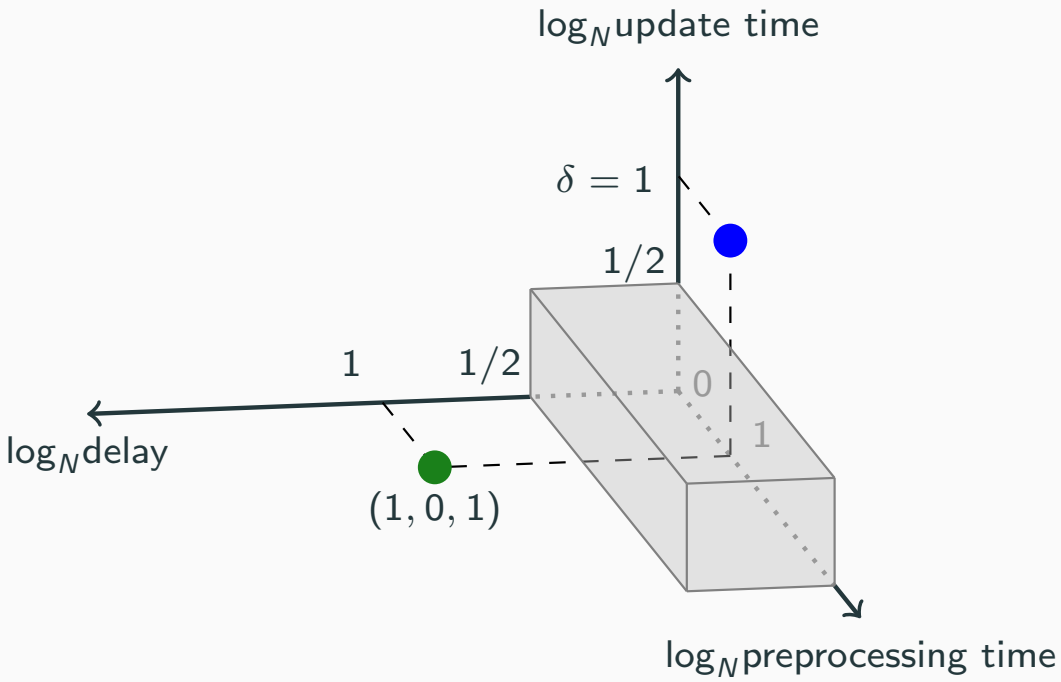

Yet, there is an algorithm that admits
sub-linear update time and sub-linear enumeration delay

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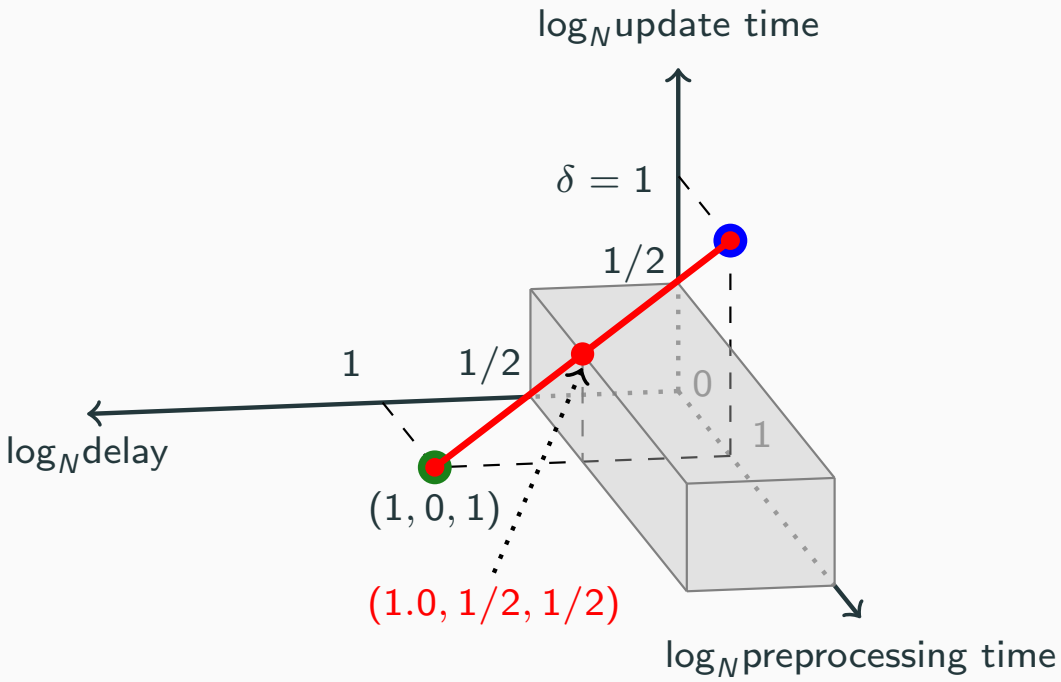


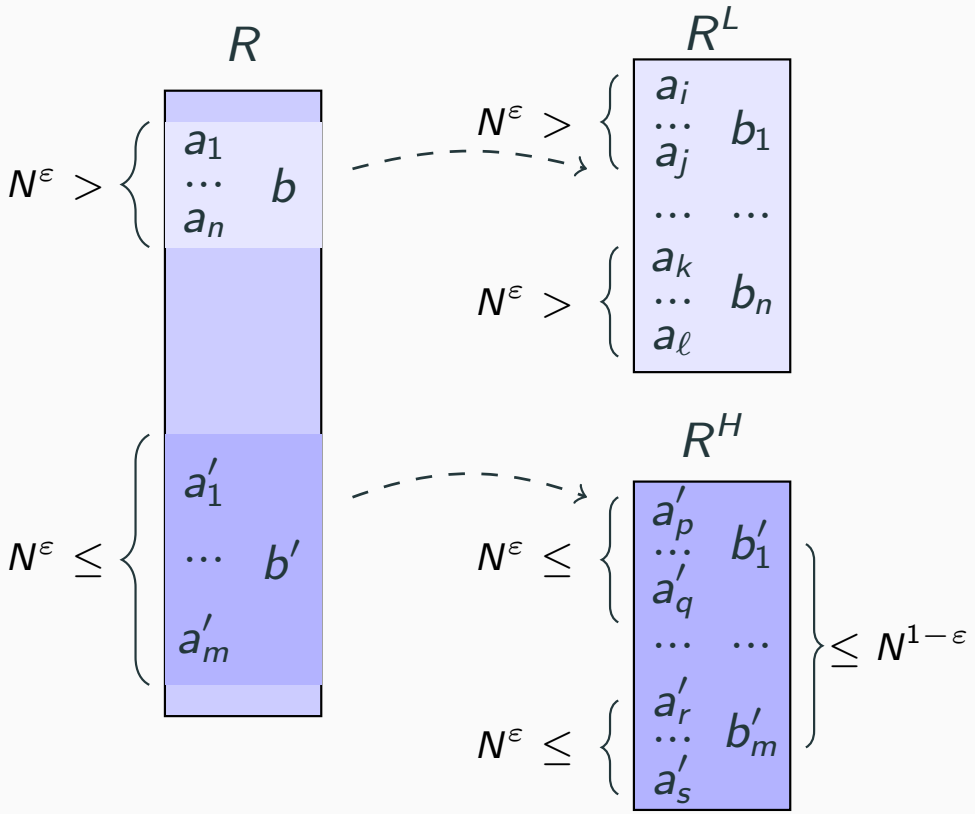
Weak Pareto optimality

Factor Partitioning

$$Q(a) = \sum_b R(a,b) \cdot S(b)$$

Partition R based on the values b into

- a light part $R^L = \{(a,b) \in R \mid |\sigma_{B=b}R| < N^\varepsilon\}$
- a heavy part $R^H = R - R^L$



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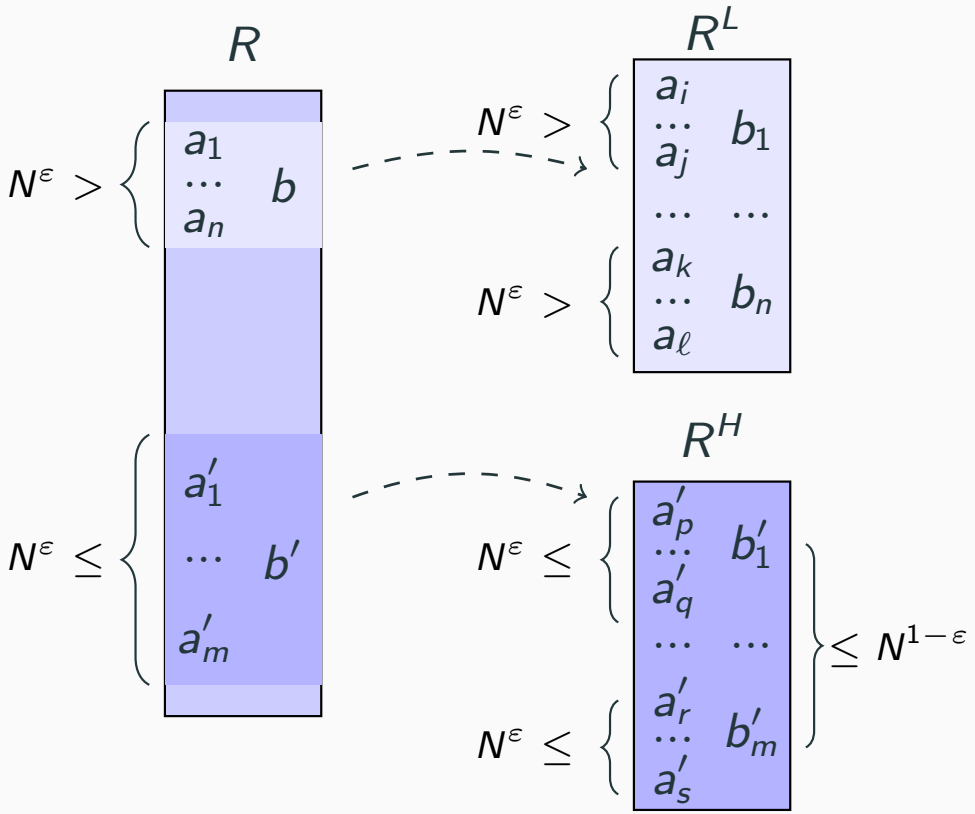

$$Q(a) = Q_L(a) + Q_H(a)$$

$$Q_L(a) = \sum_b R^L(a,b) \cdot S(b)$$

$$Q_H(a) = \sum_b R^H(a,b) \cdot S(b)$$

Light Case

$$Q_L(a) = \sum_b R^L(a, b) \cdot S(b)$$

Materialize the result

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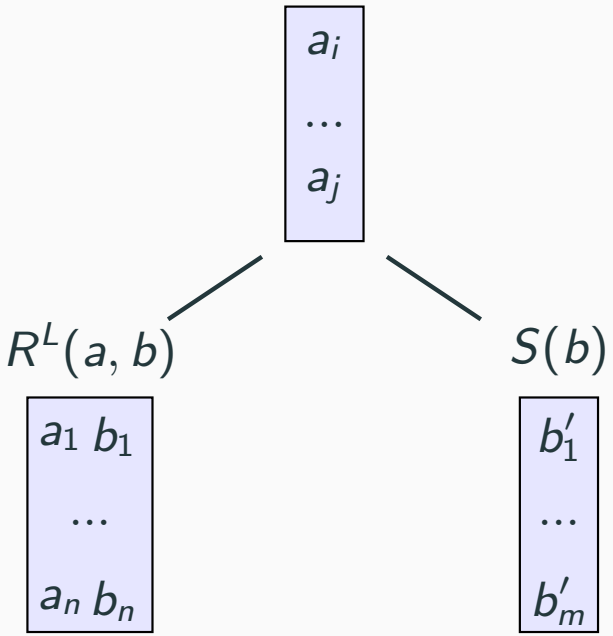

Preprocessing in the Light Case

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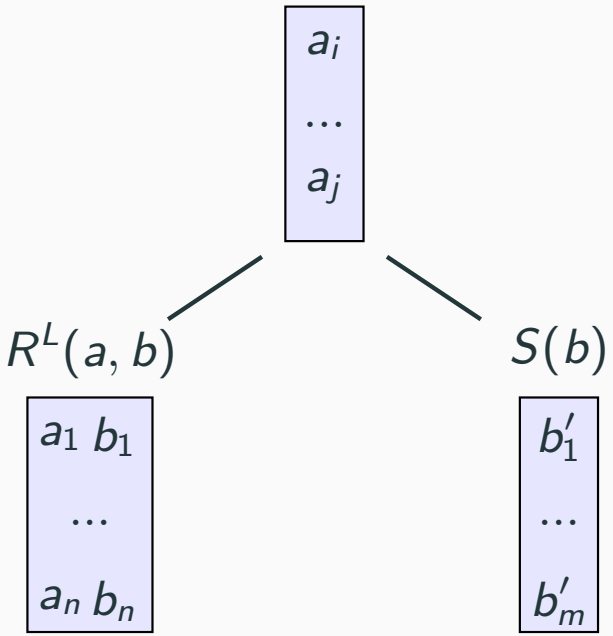
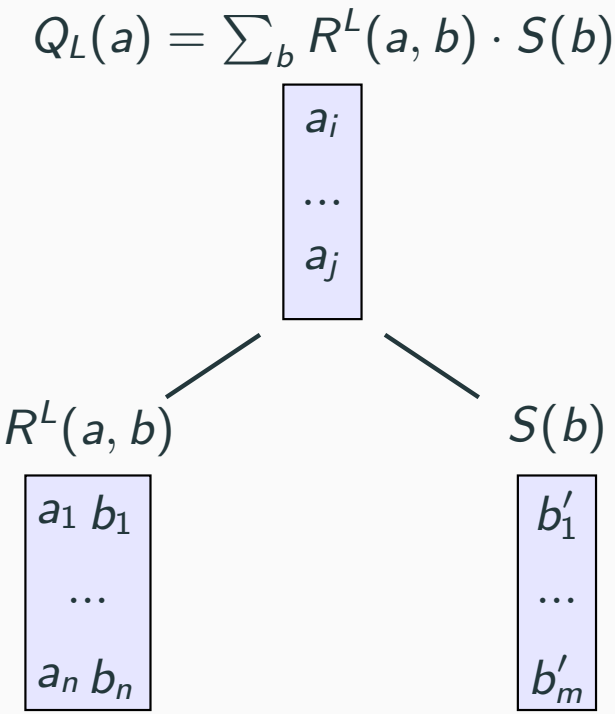


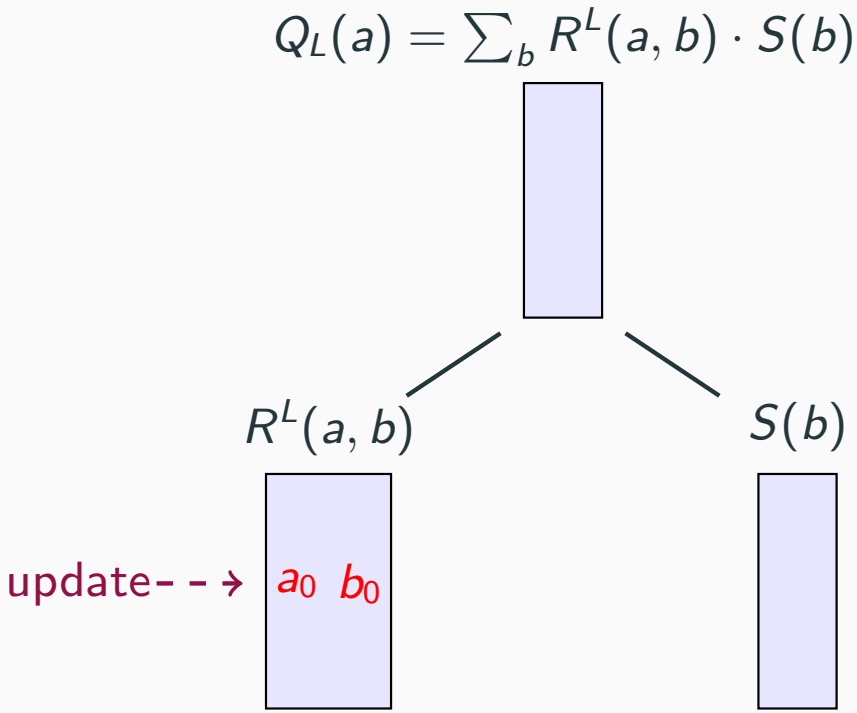
- Q_L can be computed in time $\mathcal{O}(N)$

Enumeration in the Light Case



- Q_L allows constant-time lookups and constant-delay enumeration

Updates in the Light Case



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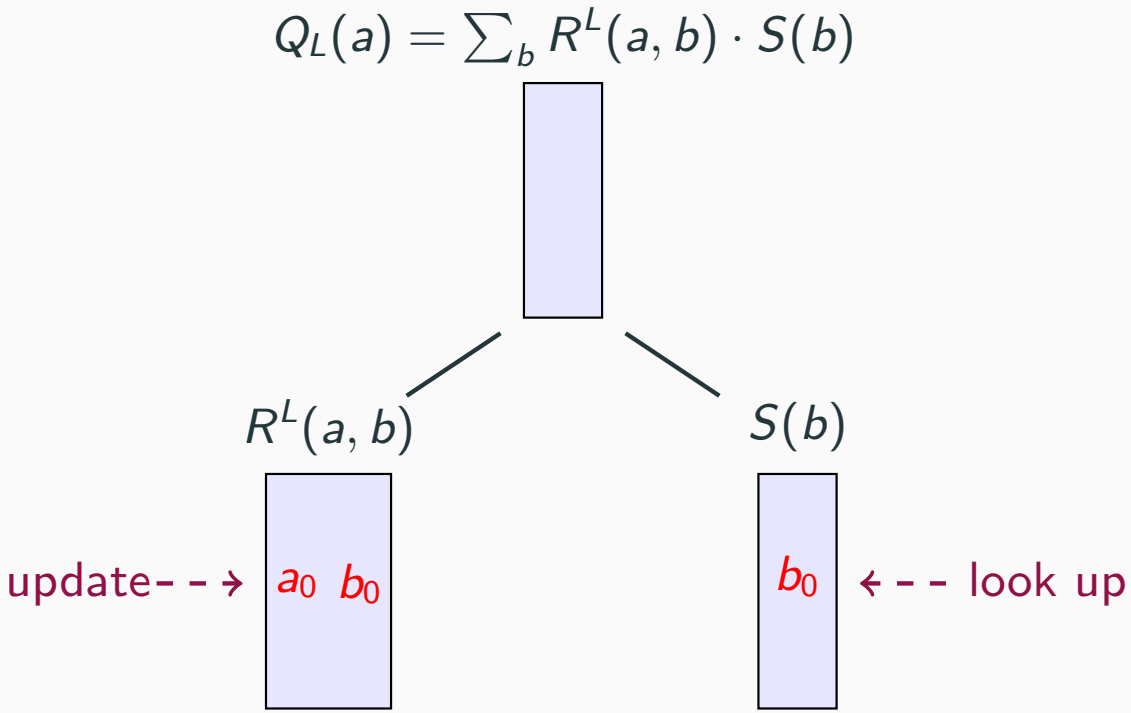

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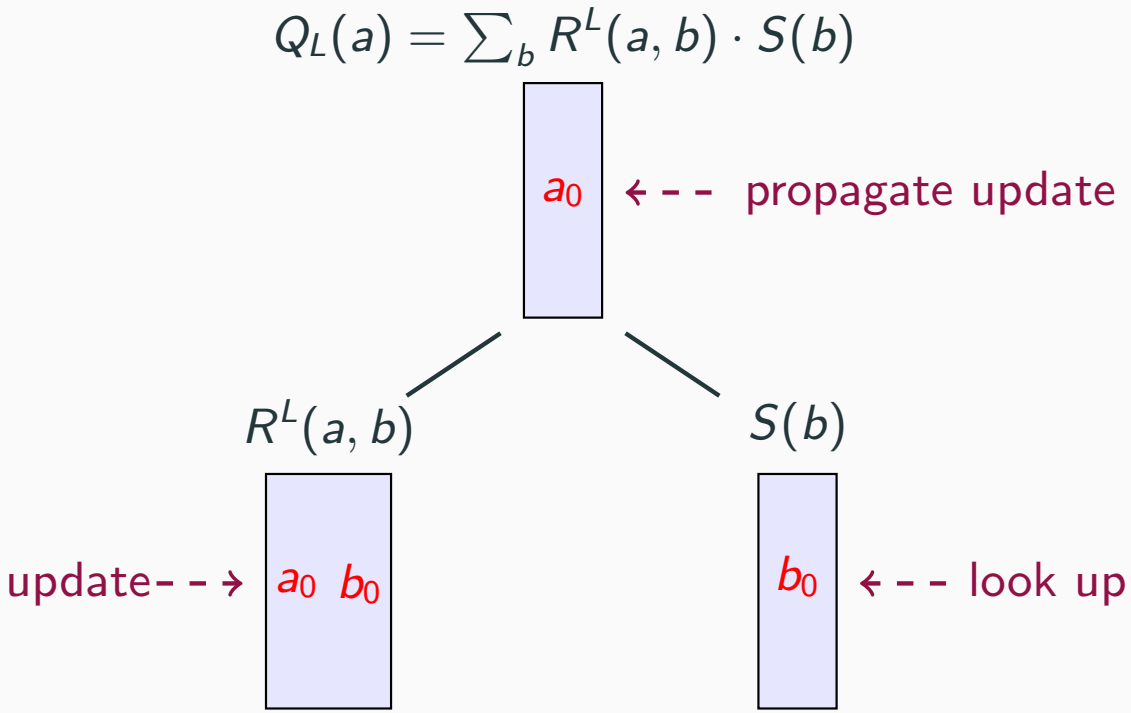

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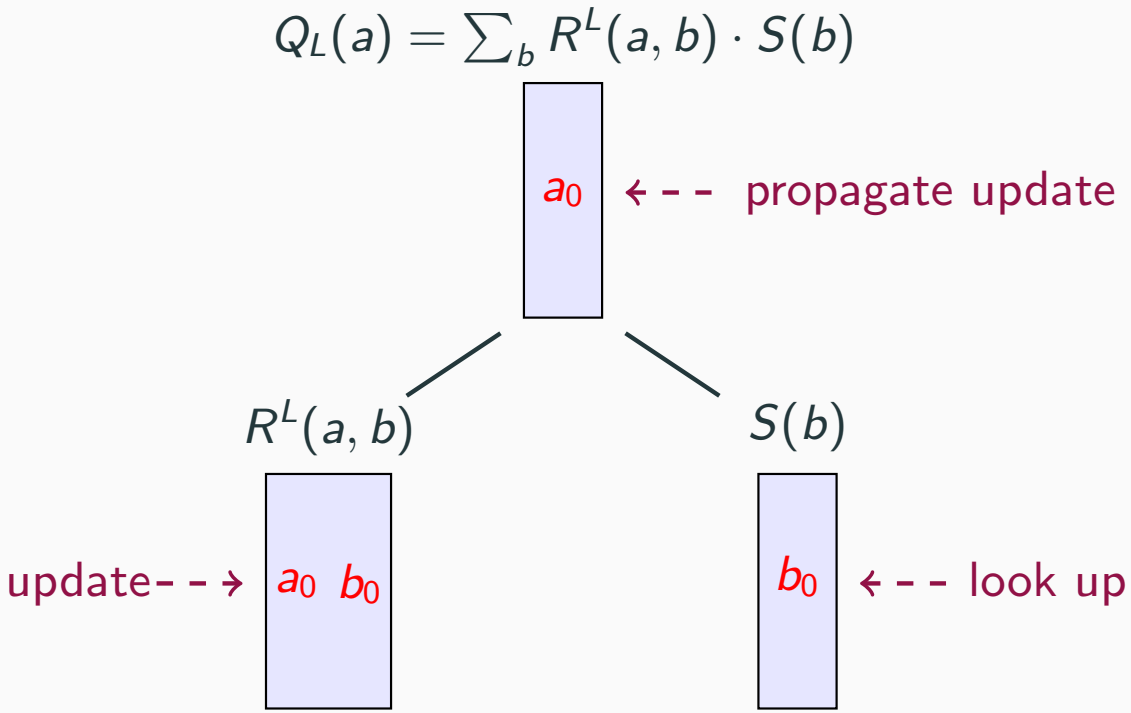

- Updates to R^L: $\mathcal{O}(1)$

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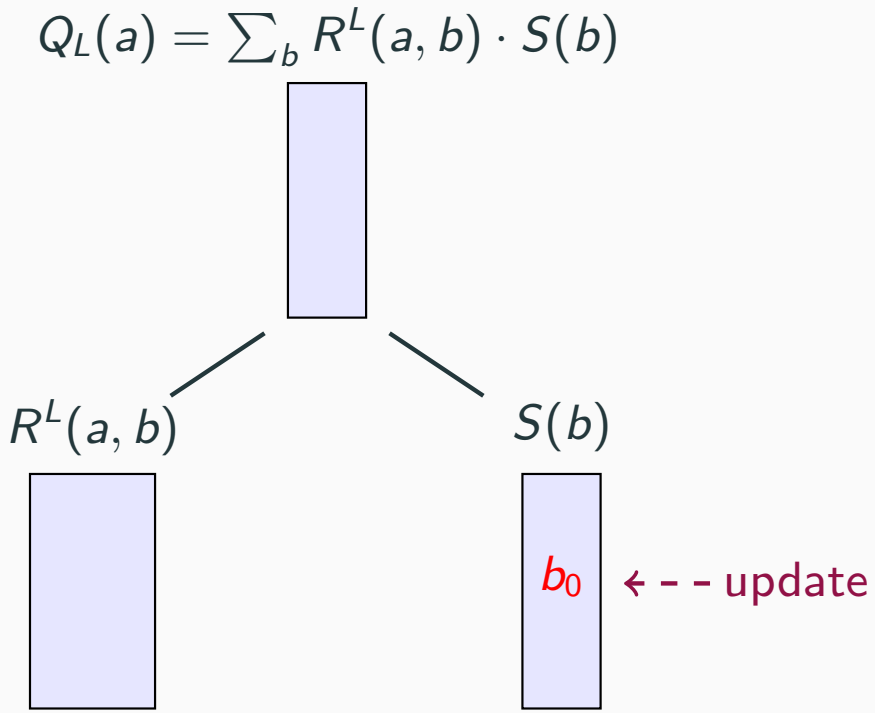

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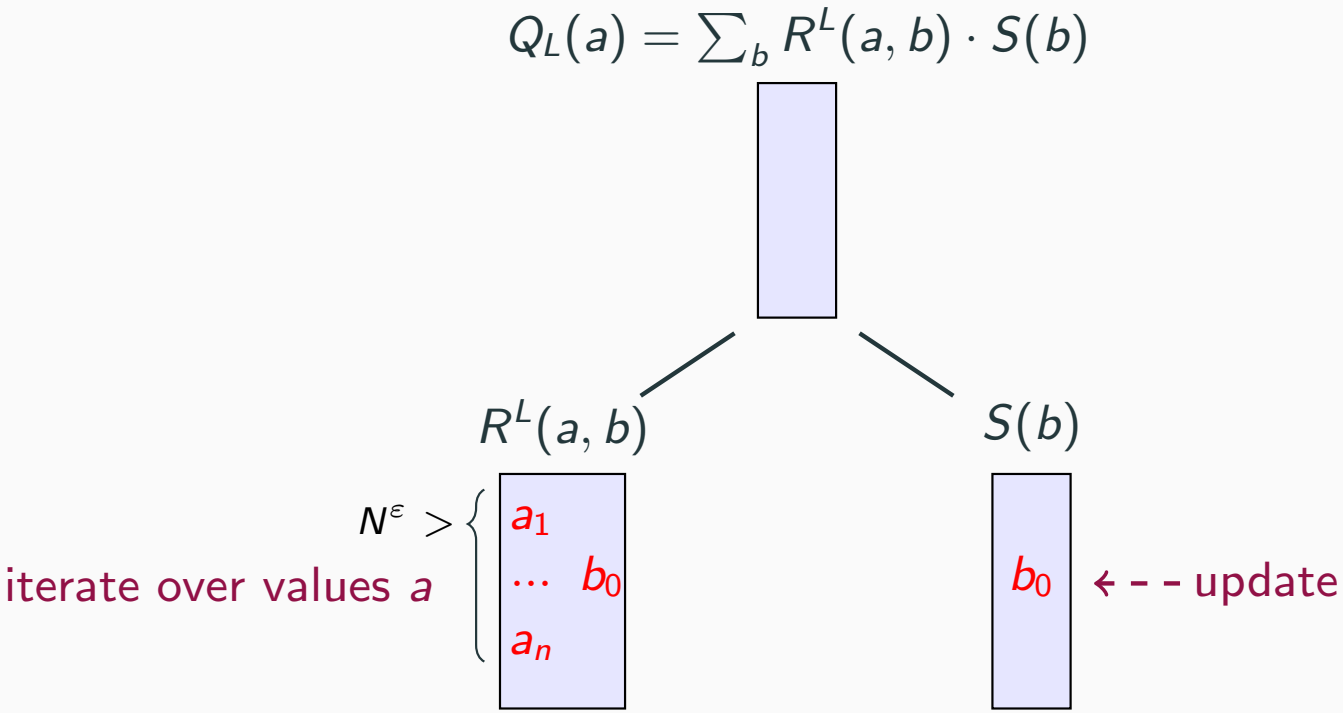

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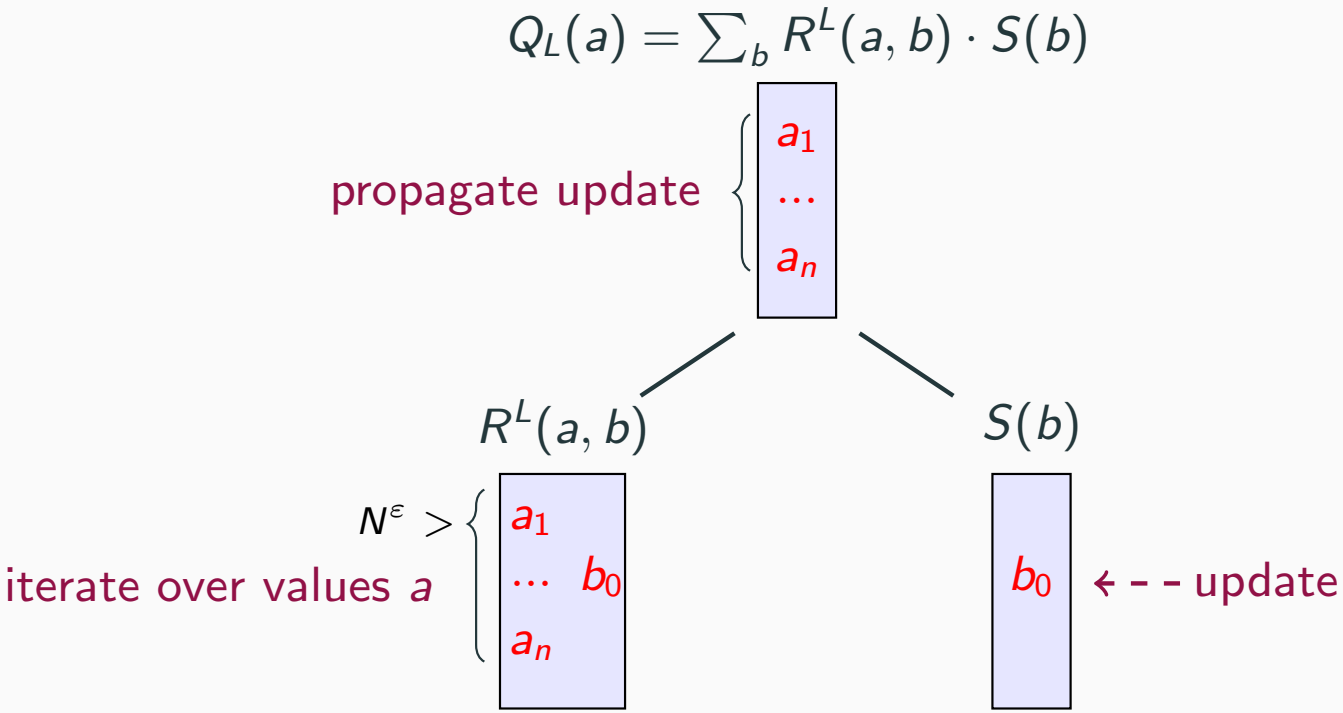

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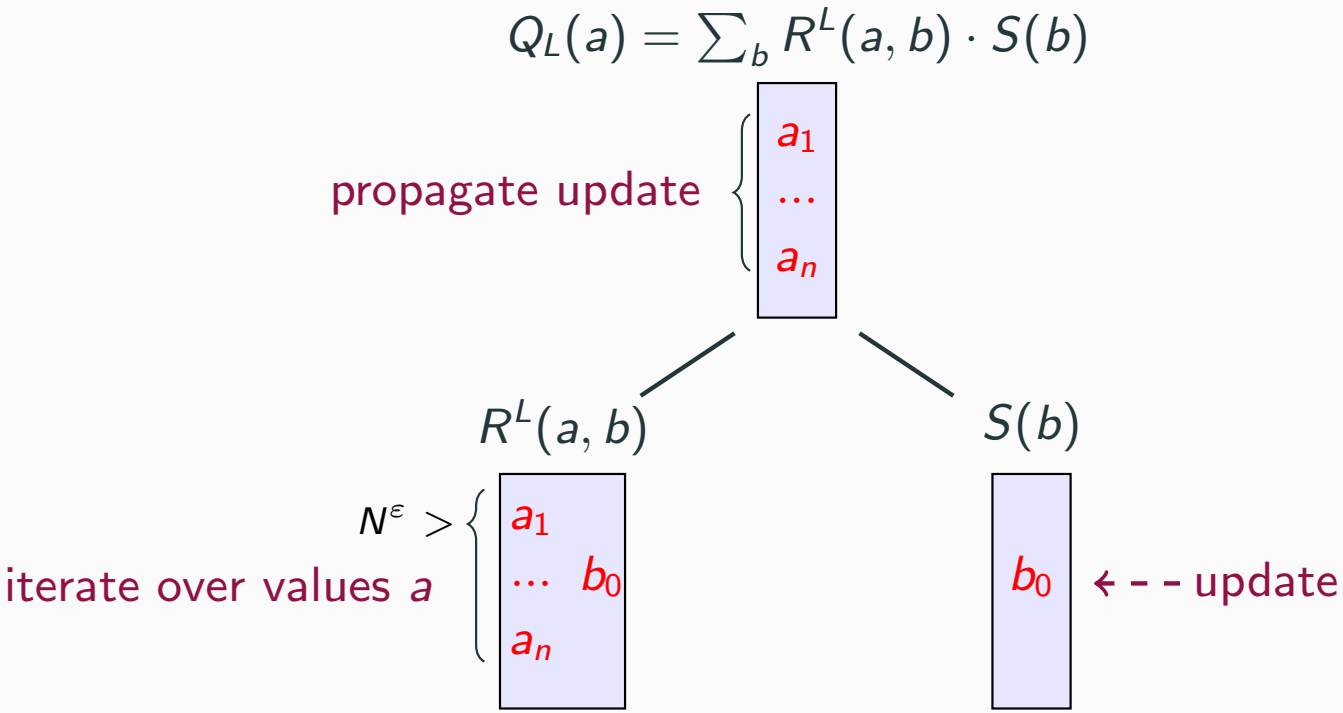

- Updates to R^L: $\mathcal{O}(1)$
- Updates to S: $\mathcal{O}(N^\varepsilon)$

Heavy Case

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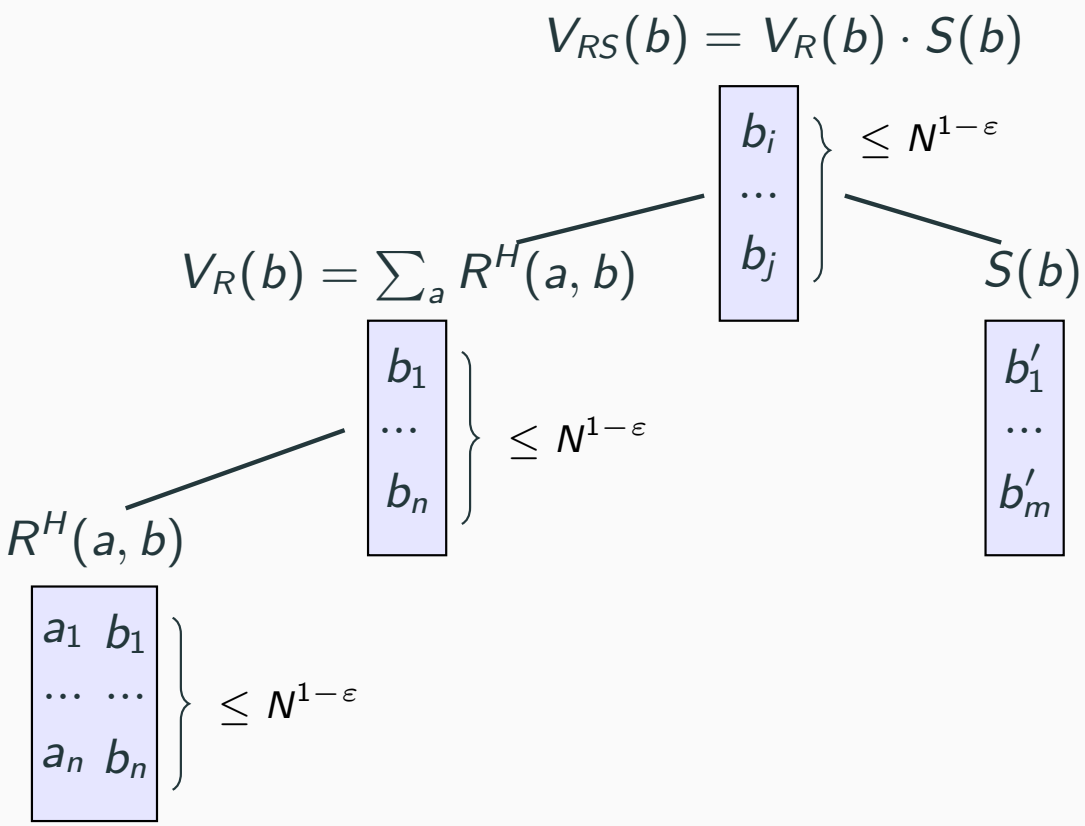

Preprocessing in the Heavy Case

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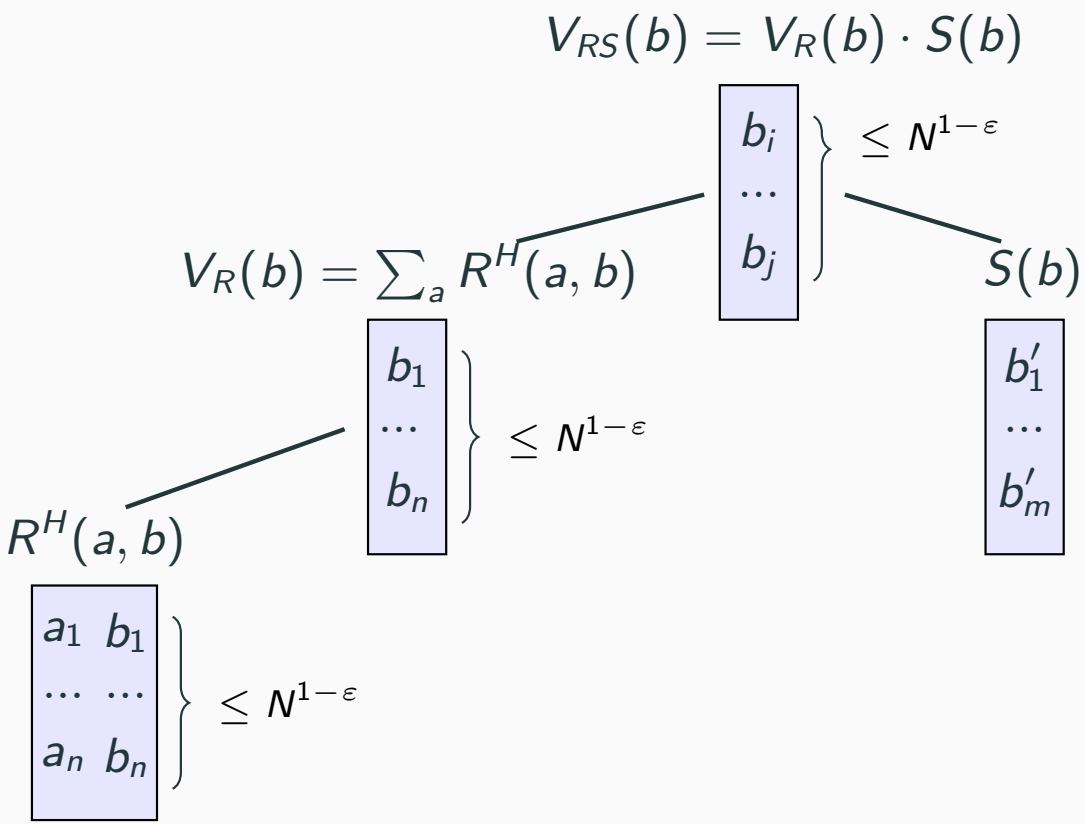
Materialize the b values in the join result



- V_{RS} can be computed in time $\mathcal{O}(N^{1-\varepsilon})$ and has at most $N^{1-\varepsilon}$ values

Enumeration in the Heavy Case

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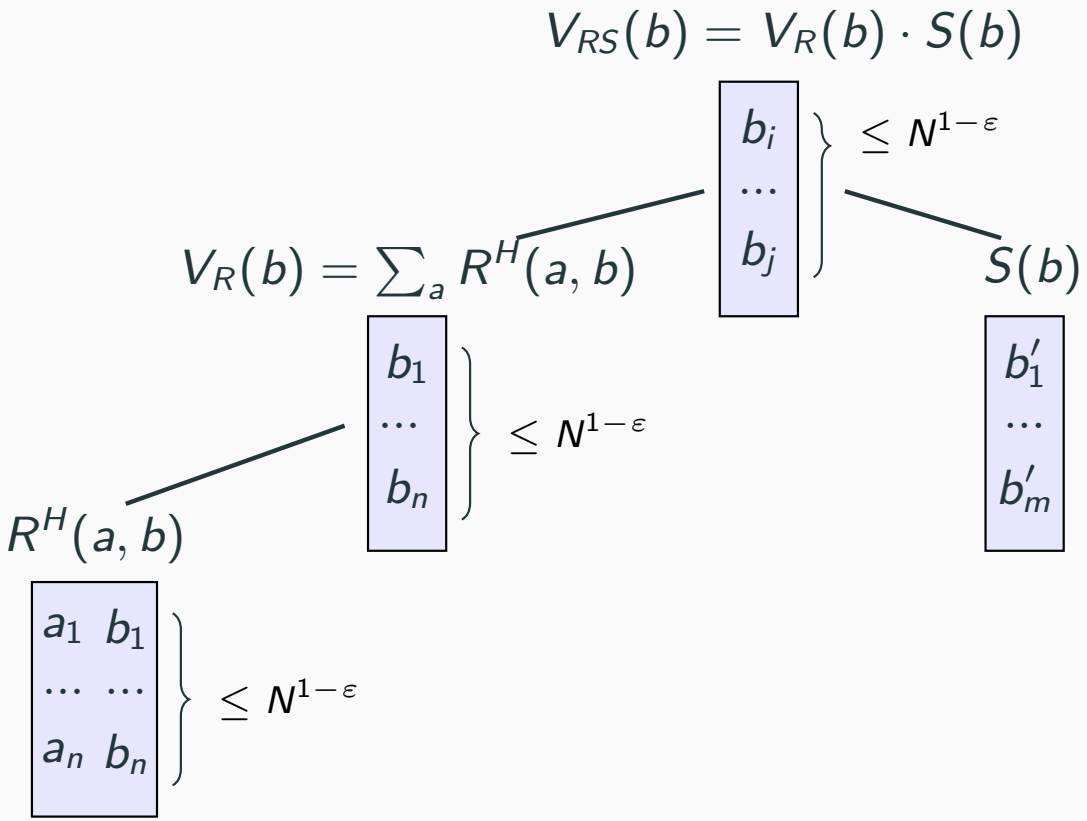

- V_{RS} contains at most $N^{1-\varepsilon}$ values b
- For each value b in V_{RS}, the values a in R^H paired with b admit constant enumeration delay

Enumeration of Distinct Tuples from Union

- $V_{RS}(b)$ contains at most $N^{1-\varepsilon}$ values
- For each value b in V_{RS}, the values a in R^H paired with b admit constant enumeration delay
- Yet: For two distinct b_1 and b_2, the sets of values a in $R^H(a, b_1)$ and $R^H(a, b_2)$ may not be disjoint

  $\Longrightarrow$ Enumerating all the values a in $R^H(a, b_1)$ and $R^H(a, b_2)$ can lead to duplicates

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Union Algorithm [CSL 2011]

- The distinct values a can be enumerated with $\mathcal{O}(N^{1-\varepsilon})$ delay

The Union Algorithm: Example

Enumeration of the distinct tuples in the union of two sets

- Both sets allow lookup time ℓ and enumeration delay d

$\Longrightarrow$ The union of the sets can be enumerated with $\mathcal{O}(\ell + d)$ delay

S_1

a_3 a_4 a_1 a_2 **EOF**

S_2

a_5 a_6 a_2 a_4 **EOF**

$S_1 \cup S_2$

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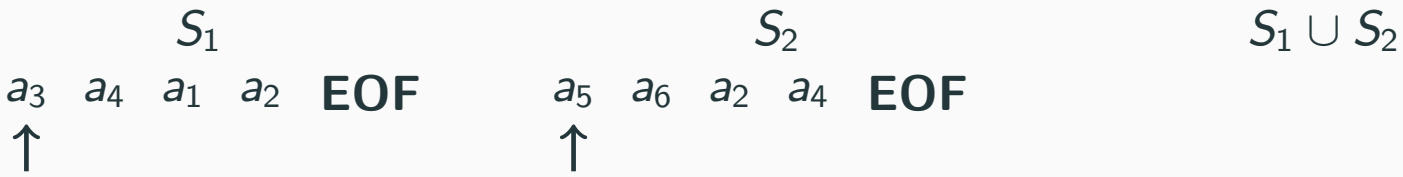

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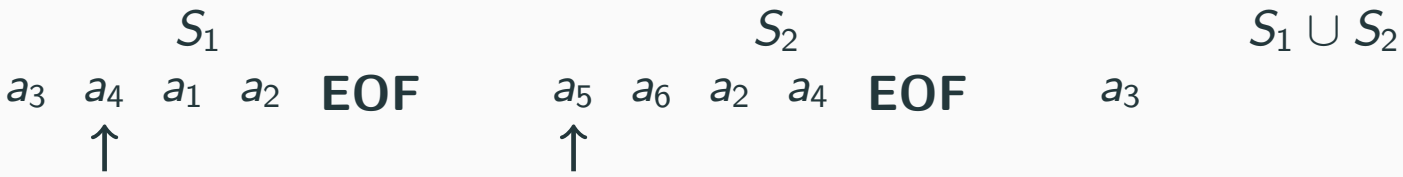

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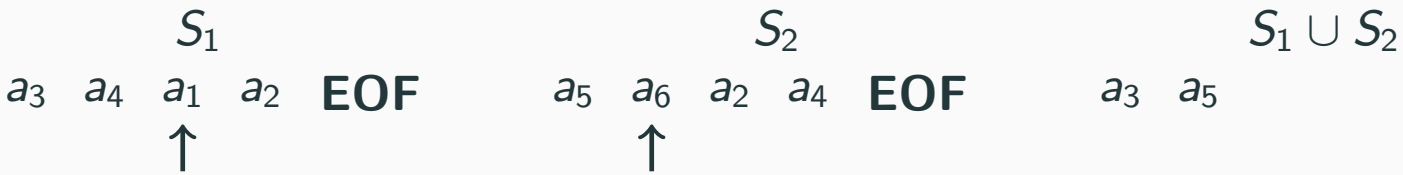

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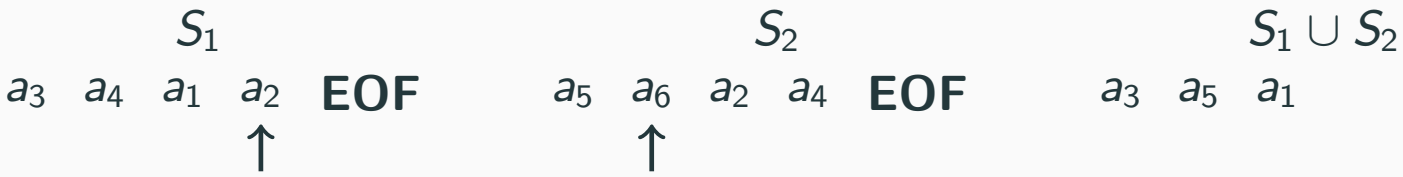

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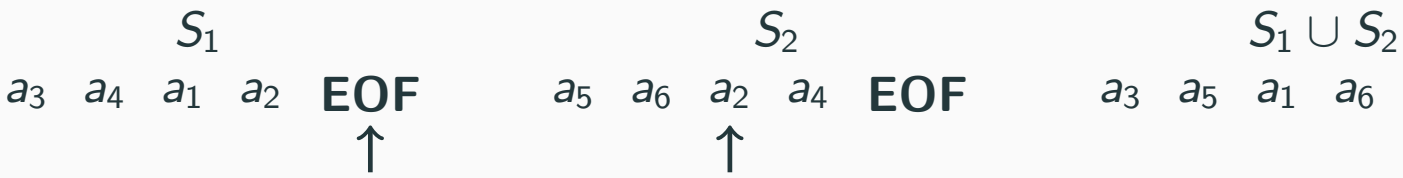

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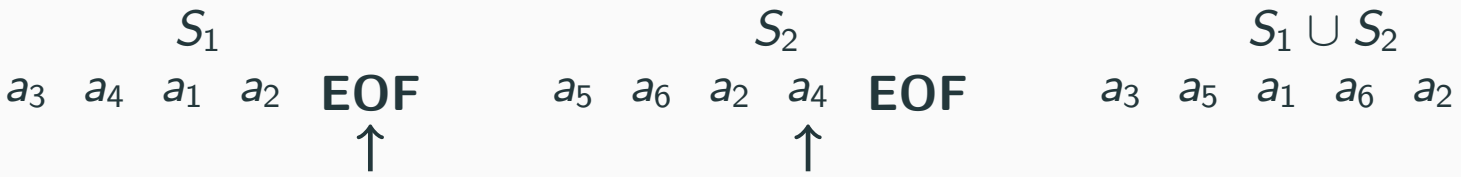

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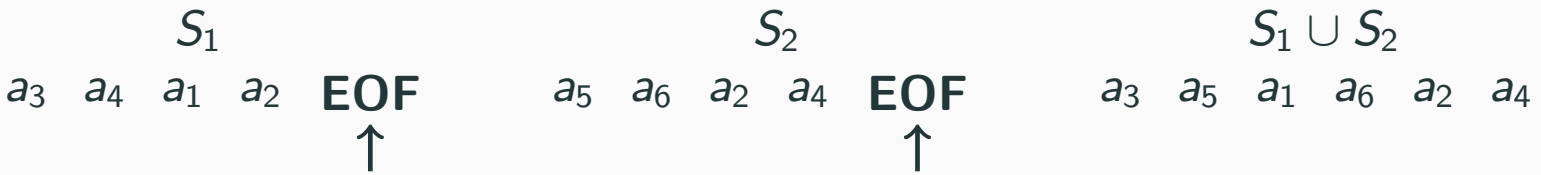

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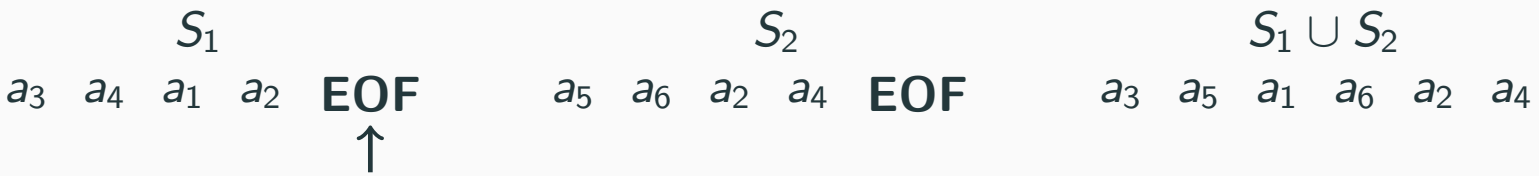
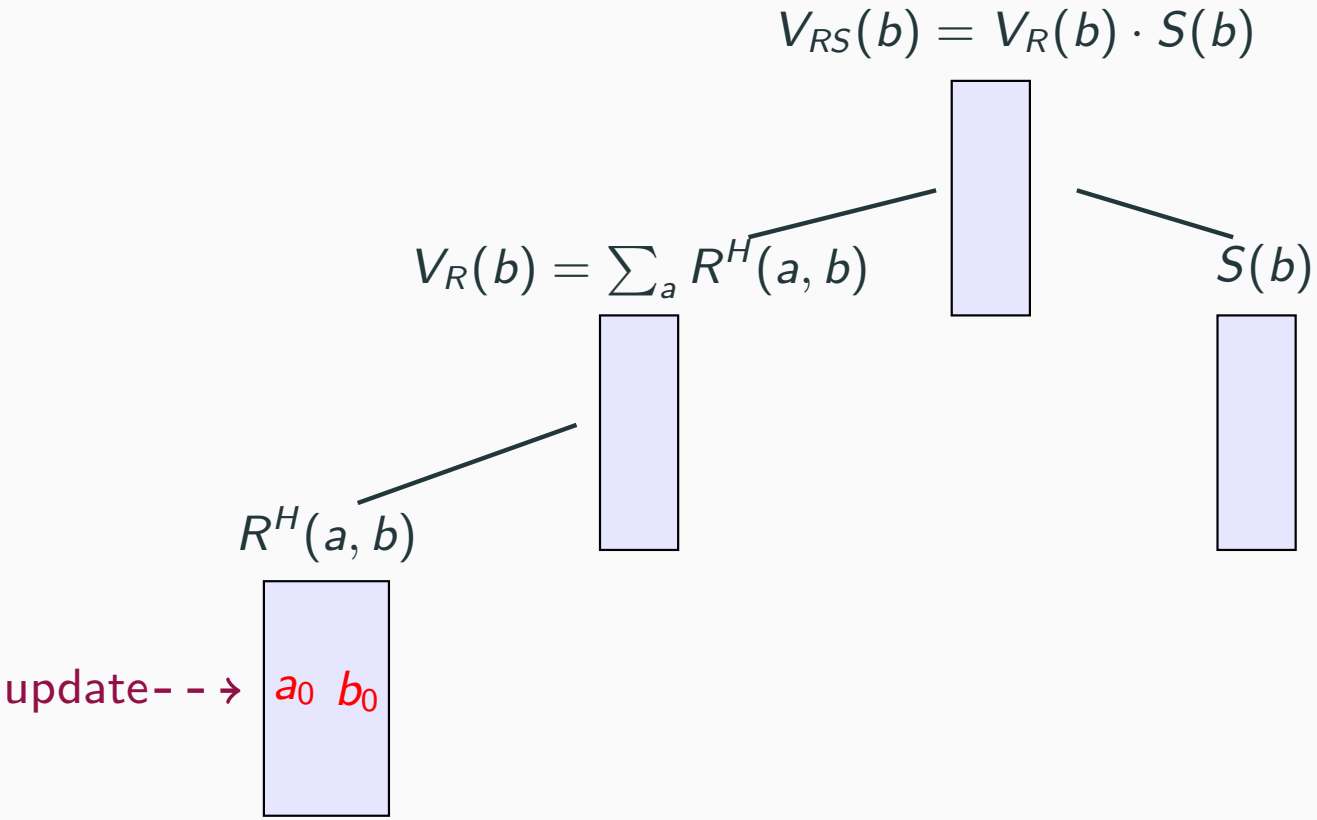


Generalization: Enumeration from the union of n sets

- Each set allows lookup time ℓ and enumeration delay d
- The union of the sets can be enumerated with $\mathcal{O}(n(\ell + d))$ delay

Updates in the Heavy Case



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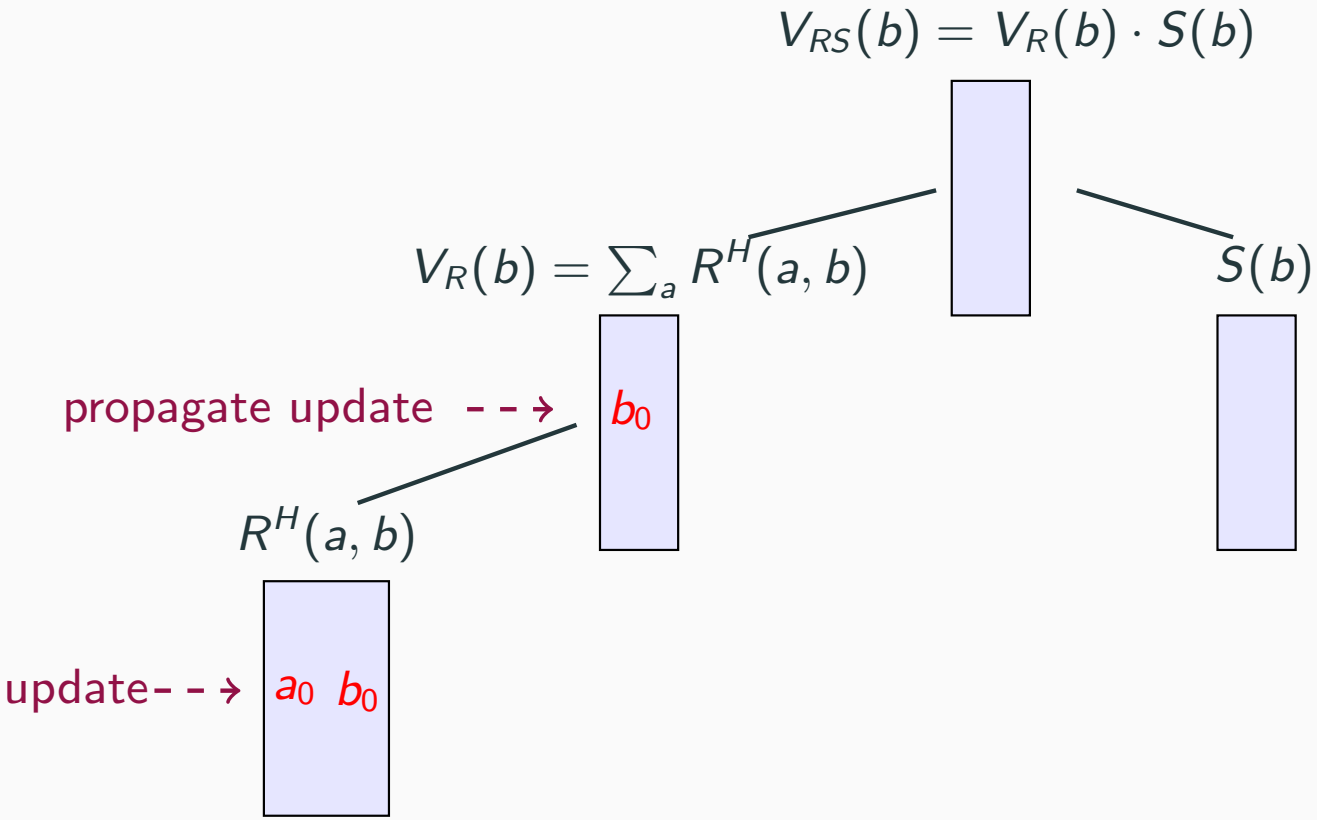

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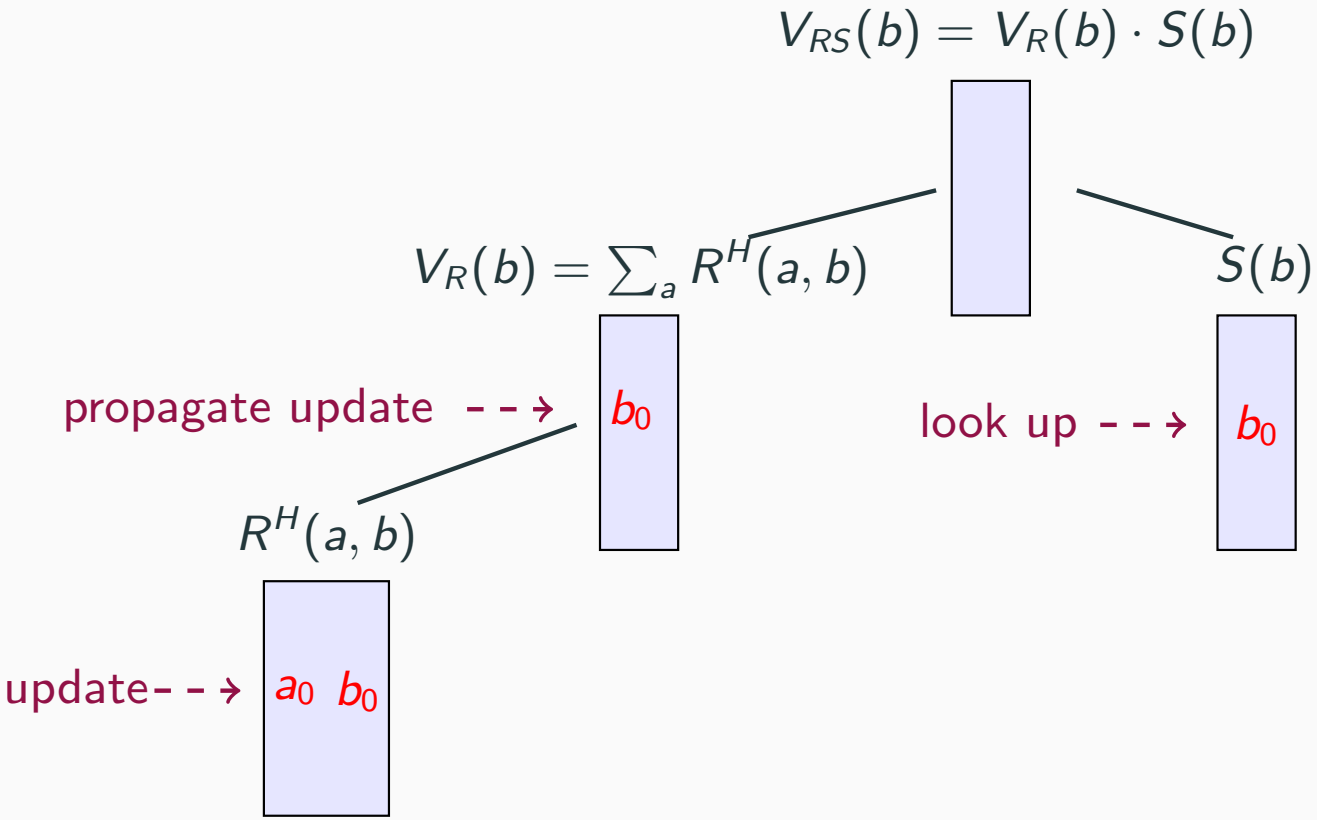

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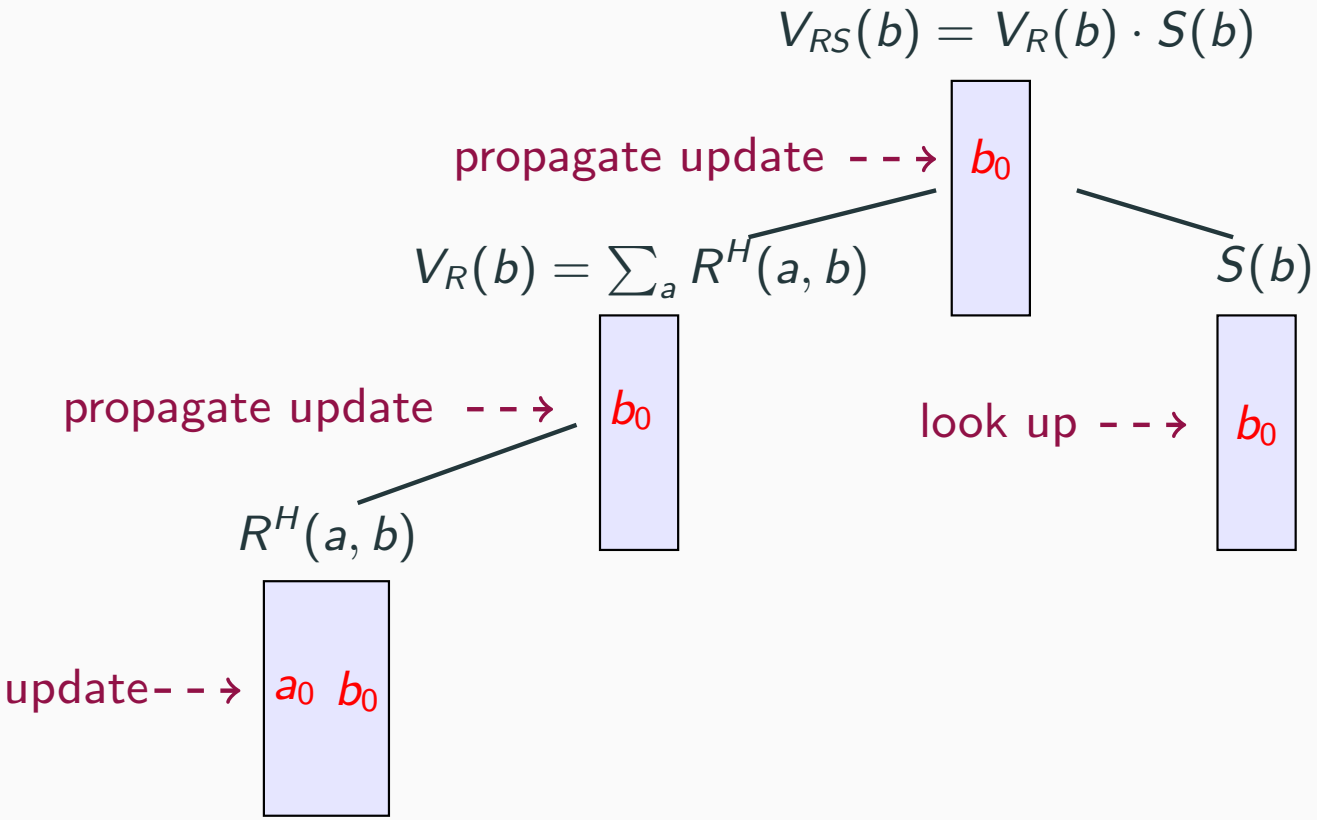

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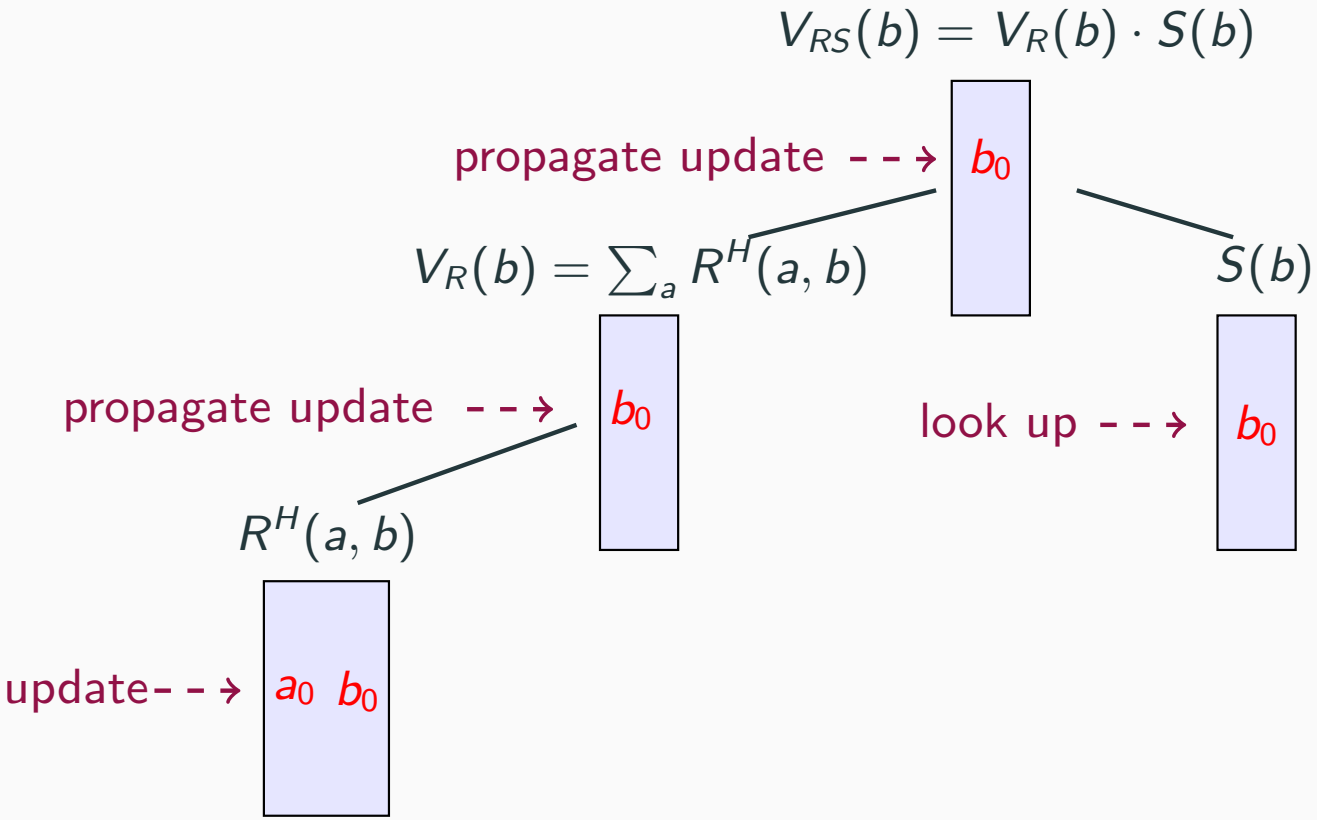

- Updates to R^H: $\mathcal{O}(1)$

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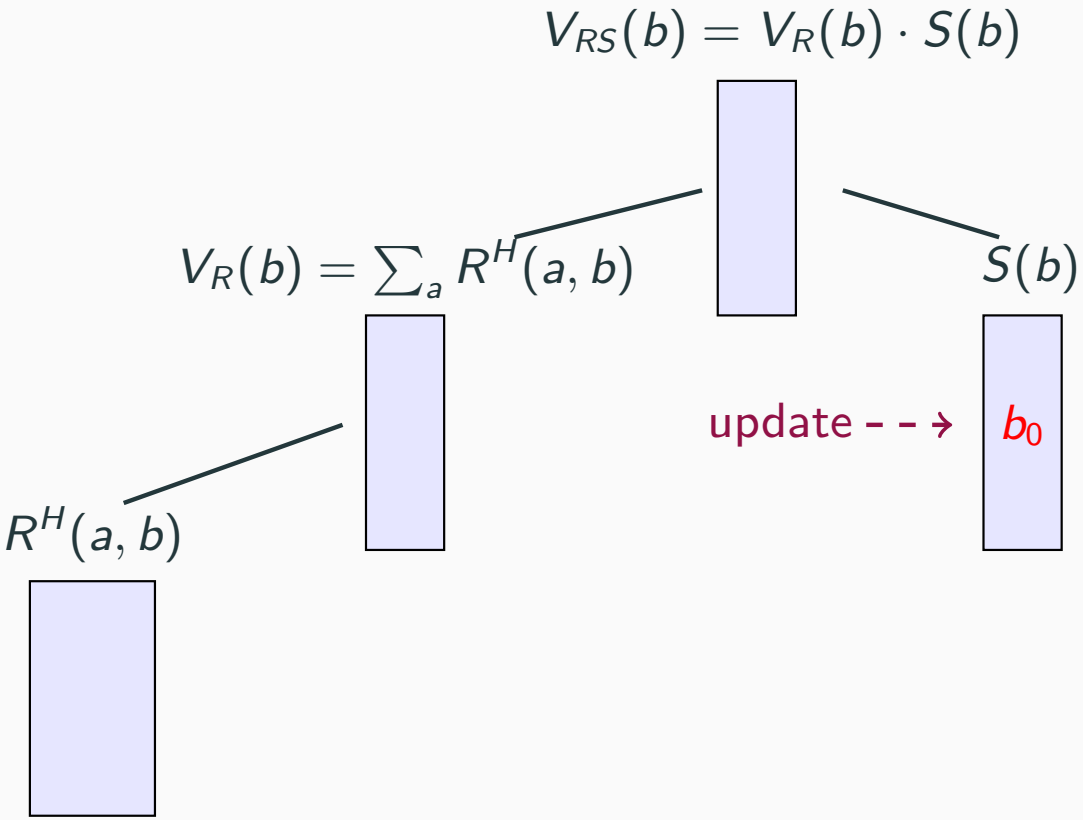

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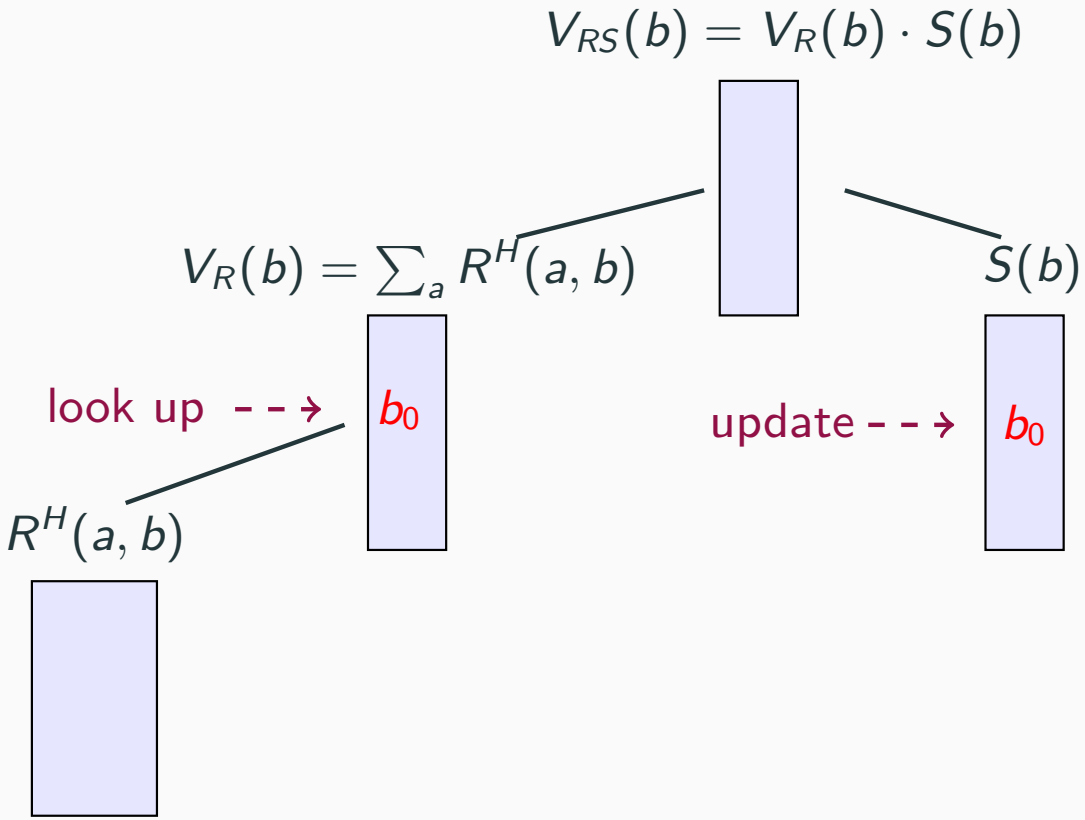

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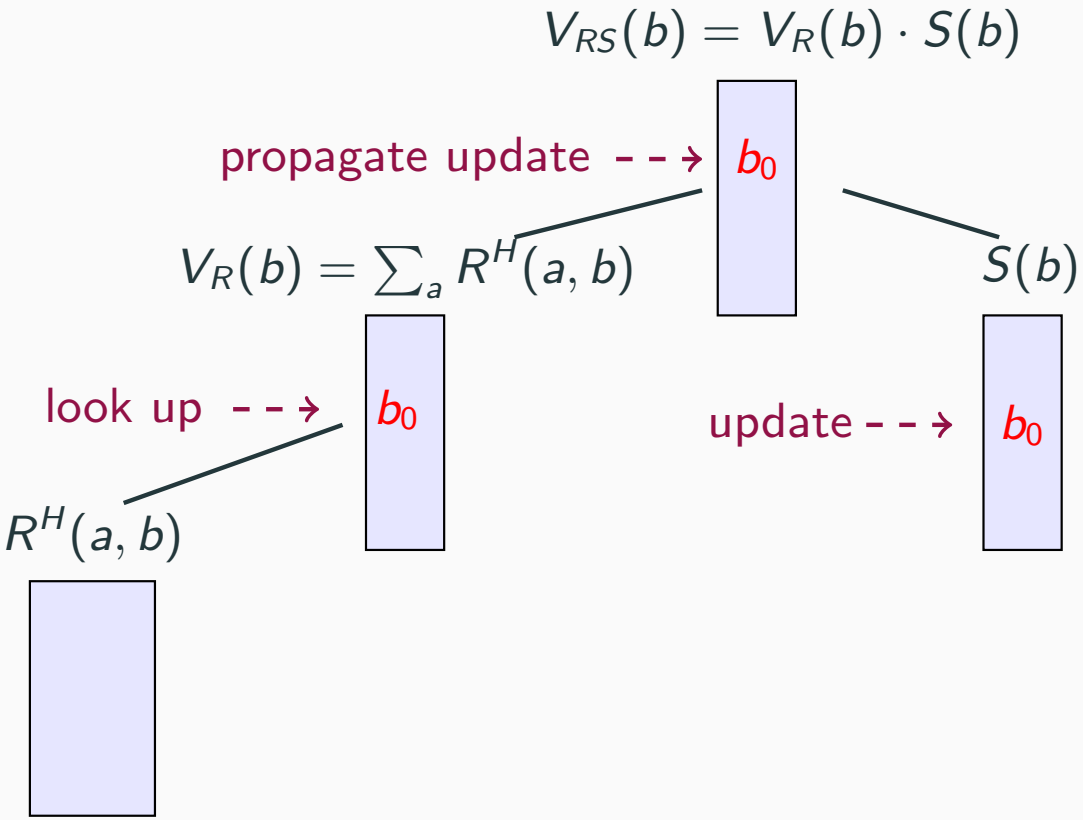

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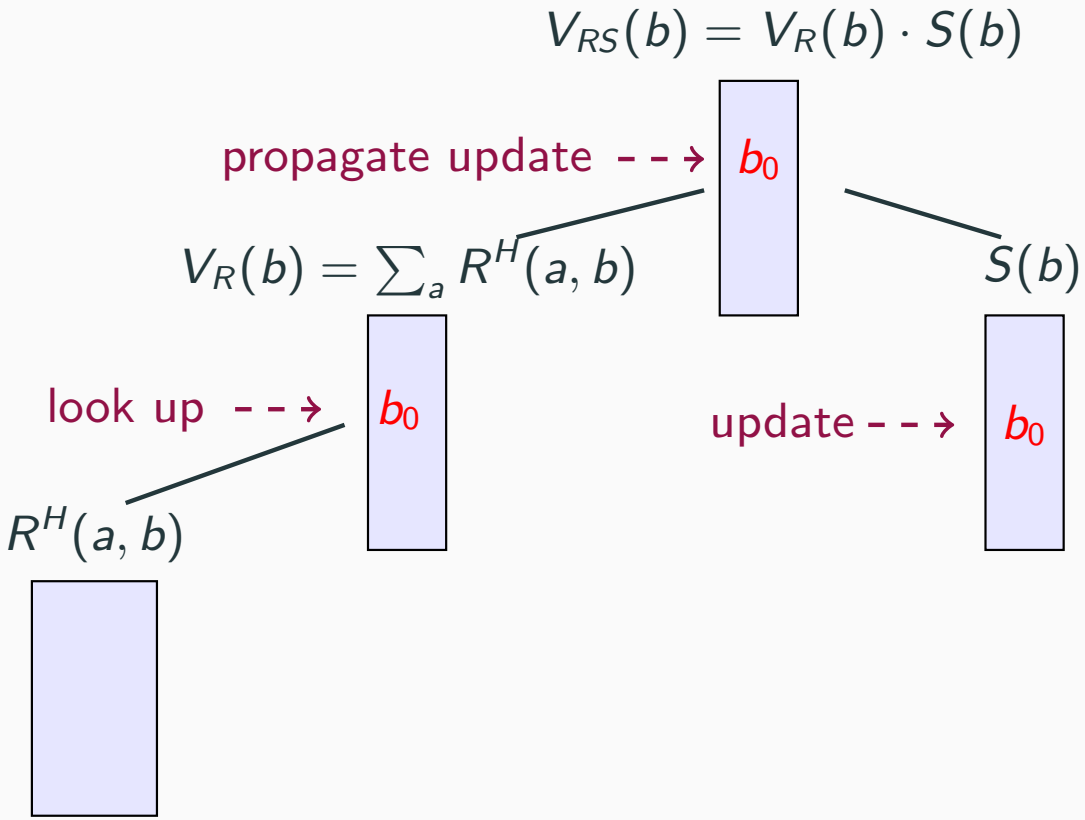


- Updates to R^H: $\mathcal{O}(1)$
- Updates to S: $\mathcal{O}(1)$

Summing Up

$$Q(a) = R(a, b) \cdot S(b)$$

Preprocessing Time

light case	heavy case	overall
$\mathcal{O}(N)$	$\mathcal{O}(N^{1-\varepsilon})$	$\mathcal{O}(N)$

Enumeration Delay

light case	heavy case	overall
$\mathcal{O}(1)$	$\mathcal{O}(N^{1-\varepsilon})$	$\mathcal{O}(N^{1-\varepsilon})$

Update Time

light case	heavy case	overall
$\mathcal{O}(N^{\varepsilon})$	$\mathcal{O}(1)$	$\mathcal{O}(N^{\varepsilon})$

Are there more queries

with the same

weak Pareto optimality

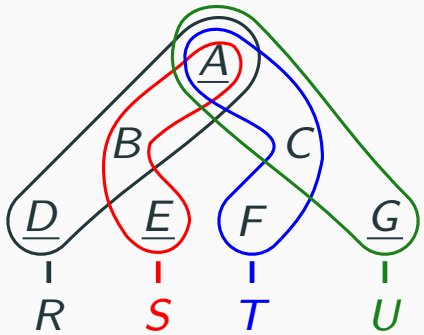
as our previous example?

δ_1-Hierarchical Queries

- For any bound variable X and any atom α of X, there is at most one other atom β so that all free variables dominated by X are covered by α and β together
- The query is hierarchical and not q-hierarchical

δ_1-hierarchical

$$Q(a, d, e, g) = R(a, b, d) \cdot S(a, b, e) \cdot T(a, c, f) \cdot U(a, c, g)$$

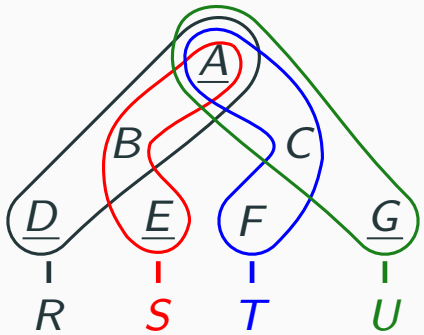


δ_1-Hierarchical Queries

- For any bound variable X and any atom α of X, there is at most one other atom β so that all free variables dominated by X are covered by α and β together
- The query is hierarchical and not q-hierarchical

δ_1-hierarchical

$Q(a, d, e, g) = R(a, b, d) \cdot S(a, b, e) \cdot T(a, c, f) \cdot U(a, c, g)$



hierarchical but not δ_1-hierarchical

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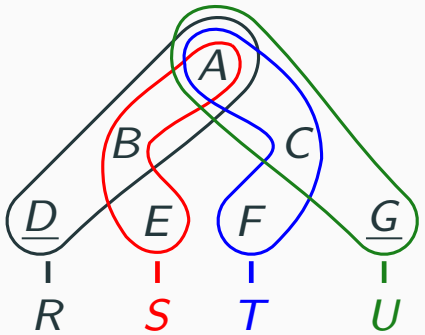


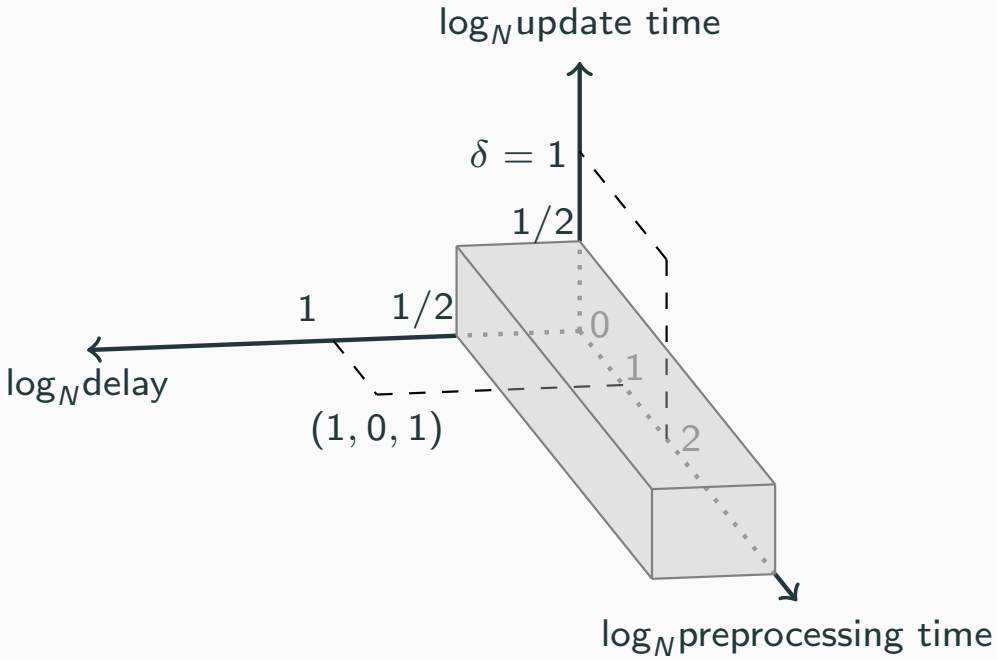
Optimality for δ_1-Hierarchical Queries

- For any δ_1-hierarchical query, there is no algorithm that admits

preprocessing time	update time	enumeration delay
arbitrary	$\mathcal{O}(N^{1/2-\gamma})$	$\mathcal{O}(N^{1/2-\gamma})$

  for any $\gamma > 0$, unless the OMv Conjecture (*) fails

(*) Online Matrix-Vector Multiplication cannot be solved in sub-cubic time



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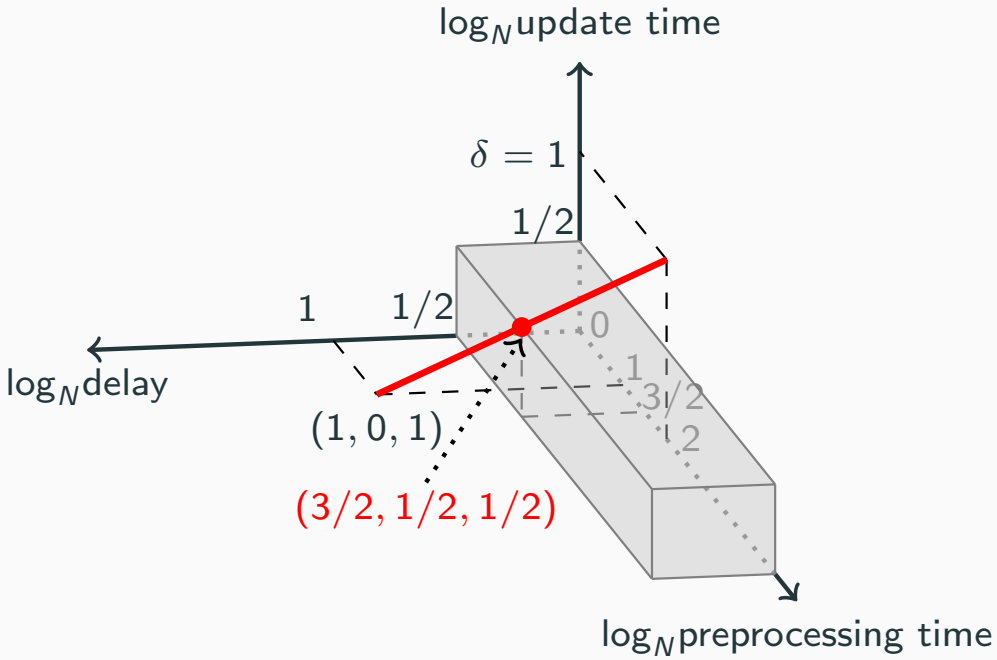
preprocessing time	update time	enumeration delay
$\mathcal{O}(N^{1+\varepsilon})$	$\mathcal{O}(N^{\varepsilon})$	$\mathcal{O}(N^{1-\varepsilon})$

(*) Online Matrix-Vector Multiplication cannot be solved in sub-cubic time



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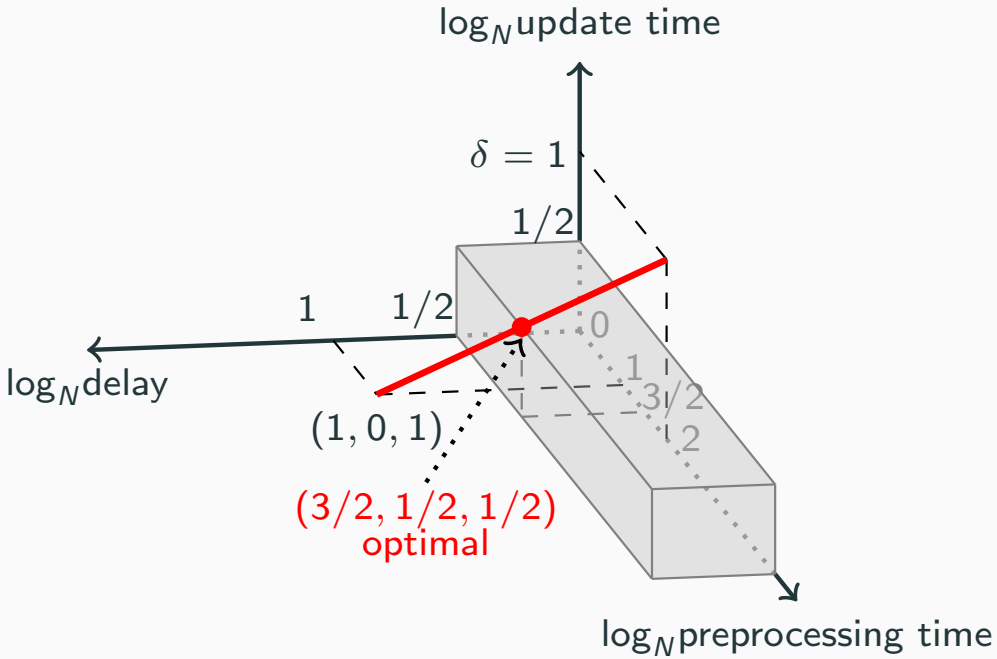
$\Longrightarrow$ For $\varepsilon = 1/2$, this is weakly Pareto optimal, unless OMv Conjecture fails

(*) Online Matrix-Vector Multiplication cannot be solved in sub-cubic time



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Thank You!