CS294-248 Special Topics in Database Theory Unit 6: Constraints, Incomplete and Probabilistic Databases (Part 2)

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Outline

- Tuesday: Generalized Constraints, Semantics Optimization.

- Today: Repairs, Incomplete Databases

Recap: Generalized Dependencies

Tuple-Generating Dependency (TGD):

$$\forall \boldsymbol{x}\,(A_1 \wedge \ldots \wedge A_m \Rightarrow \exists \boldsymbol{y}(B_1 \wedge \cdots \wedge B_k))$$

The TGD is full if there is no $\exists \boldsymbol{y}$

Equality-Generating Dependency (EGD):

$$\forall \boldsymbol{x}(A_1 \wedge \ldots \wedge A_m \Rightarrow x_i = x_j)$$

Recap: Chase

Given $\theta : A \to Q$, a chase step is $Q \stackrel{\sigma,\theta}{\to} Q'$, where

- If $\sigma \equiv \forall \boldsymbol{x}(A \Rightarrow \exists \boldsymbol{y} B)$, then $Q' = Q \wedge \theta(B)$.
- If $\sigma \equiv \forall \boldsymbol{x}(A \Rightarrow (x_i = x_j))$, then $Q' = Q[x_j/x_i]$.

Key property: $\sigma \models Q \equiv Q'$.

Repairs for FDs

Definition

Consider a set of constraints Σ and a database $\boldsymbol{D}$.

$\boldsymbol{D} \not\models \Sigma$.

The Database Repair Problem

Find another database $\boldsymbol{D}'$ such that $\boldsymbol{D}' \models \Sigma$ and $|\boldsymbol{D} \Delta \boldsymbol{D'}|$ is minimal.

(Recall: $S_1 \Delta S_2 = (S_1 - S_2) \cup (S_2 - S_1)$.)

Equivalently: perform a minimum number of updates to satisfy Σ.

The FD-Repair Problem

Σ is a set of FDs

The updates are restricted to be be deletions

Given $\boldsymbol{D}$, delete minimum number of tuples to obtain $\boldsymbol{D}' \subseteq \boldsymbol{D}$ and $\boldsymbol{D}' \models \Sigma$.

We study the complexity as a function of $|\boldsymbol{D}|$
following [Livshits et al., 2020].

Example 1: Repairing $A \rightarrow B$

$A \rightarrow B$

A	B	C	D
a_1	b_1	c_1	$\cdots$
a_1	b_2	c_1	$\cdots$
a_1	b_2	c_2	$\cdots$
a_2	b_1	c_1	$\cdots$
a_2	b_1	c_2	$\cdots$
a_2	b_2	c_3	$\cdots$
a_3	$\cdots$	$\cdots$	$\cdots$
	$\cdots$		

Compute optimal repair. How?

Example 1: Repairing $A \to B$

$A \to B$

A	B	C	D
a_1	b_1	c_1	$\cdots$
a_1	b_2	c_1	$\cdots$
a_1	b_2	c_2	$\cdots$
a_2	b_1	c_1	$\cdots$
a_2	b_1	c_2	$\cdots$
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a_2	b_1	c_1	$\cdots$
a_2	b_1	c_2	$\cdots$
a_2	b_2	c_3	$\cdots$
a_3	$\cdots$	$\cdots$	$\cdots$
	$\cdots$		

Group the tuples by A

In each group $a_1, a_2, \ldots$ keep only one b_j (the most frequent).

Example 1: Repairing $A \to BC$

$A \to BC$

A	B	C	D
a_1	b_1	c_1	$\cdots$
a_1	b_2	c_1	$\cdots$
a_1	b_2	c_2	$\cdots$
a_2	b_1	c_1	$\cdots$
a_2	b_1	c_2	$\cdots$
a_2	b_2	c_3	$\cdots$
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a_2	b_1	c_1	$\cdots$
a_2	b_1	c_2	$\cdots$
a_2	b_2	c_3	$\cdots$
a_3	$\cdots$	$\cdots$	$\cdots$
	$\cdots$		

Compute optimal repair. How?

Same as before: treat BC as a single attribute.

Example 3: $A \to B \to C$

$A \to B \to C$

A	B	C	D
a_1	b_1	c_1	$\cdots$
a_1	b_2	c_1	$\cdots$
a_1	b_2	c_2	$\cdots$
a_2	b_1	c_1	$\cdots$
a_2	b_1	c_2	$\cdots$
a_2	b_2	c_3	$\cdots$
a_3	$\cdots$	$\cdots$	$\cdots$
	$\cdots$		

Compute optimal repair. How?

Example 3: $A \to B \to C$

$A \to B \to C$

A	B	C	D
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a_1	b_2	c_1	$\cdots$
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a_2	b_1	c_1	$\cdots$
a_2	b_1	c_2	$\cdots$
a_2	b_2	c_3	$\cdots$
a_3	$\cdots$	$\cdots$	$\cdots$
	$\cdots$		

Compute optimal repair. How?

This is NP-hard!

Reduction from Max-SAT

Theorem ([Williams, 2016])

The problem given a 2CNF, check $\geq 7/10$ clauses can be satisfied is NP-complete.

Proof for $A \to B \to C$

Start with a 2CNF formula $\Phi = C_1 \wedge C_2 \wedge \cdots \wedge C_n$

Create a relation instance $R(A, B, C)$ as follows:

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Create a relation instance $R(A, B, C)$ as follows:

For each clause $C_i = ((\neg)X \vee (\neg)Y)$ add two tuples to R

- Tuple $(i, X, 0)$ or $(i, X, 1)$, depending on whether $\neg X$ or X
- Tuple $(i, Y, 0)$ or $(i, Y, 1)$, depending on whether $\neg Y$ or Y

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Claim $\geq 7n/10$ clauses can be satisfied iff $\exists$ repair of size $\geq 7n/10$.

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Claim $\geq 7n/10$ clauses can be satisfied iff $\exists$ repair of size $\geq 7n/10$.

Proof $A \to B$ ensures that we retain ≤ 1 tuple per clause

$B \to C$ ensures that we assign consistent values to the same variable.

Discussion so Far

$A \to B$ in PTIME

$A \to BC$ in PTIME

$A \to B \to C$ NP-hard

What's the general rule?

Unusual FDs

We are familiar with $AB \to CD$ or $A \to C$.

What does $A \to \emptyset$ mean?

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It is always true.

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Unusual FDs

We are familiar with $AB \to CD$ or $A \to C$.

What does $A \to \emptyset$ mean? It is always true.

What does $\emptyset \to A$ mean? A has a single value.

Example 4: $\emptyset \to A$

$\boxed{\emptyset \to A}$

A	B	C	D
a_1	b_1	c_1	$\cdots$
a_1	b_2	c_1	$\cdots$
a_1	b_2	c_2	$\cdots$
a_2	b_1	c_1	$\cdots$
a_2	b_1	c_2	$\cdots$
a_2	b_2	c_3	$\cdots$
a_3	$\cdots$	$\cdots$	$\cdots$
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Compute optimal repair. How?

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$\emptyset \to A$

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a_1	b_2	c_2	$\cdots$
a_2	b_1	c_1	$\cdots$
a_2	b_1	c_2	$\cdots$
a_2	b_2	c_3	$\cdots$
a_3	$\cdots$	$\cdots$	$\cdots$
	$\cdots$		

Compute optimal repair. How?

We keep a single value of A, namely the most frequent one.

Example 4: $\emptyset \to A$

$\emptyset \to A$

A	B	C	D
a_1	b_1	c_1	$\cdots$
a_1	b_2	c_1	$\cdots$
a_1	b_2	c_2	$\cdots$
a_2	b_1	c_1	$\cdots$
a_2	b_1	c_2	$\cdots$
a_2	b_2	c_3	$\cdots$
a_3	$\cdots$	$\cdots$	$\cdots$
	$\cdots$		

Compute optimal repair. How?

We keep a single value of A, namely the most frequent one.

Now consider:

$\emptyset \to A$
$B \to C$

Compute optimal repair. How?

Example 4: $\emptyset \to A$

$\emptyset \to A$

A	B	C	D
a_1	b_1	c_1	$\cdots$
a_1	b_2	c_1	$\cdots$
a_1	b_2	c_2	$\cdots$
a_2	b_1	c_1	$\cdots$
a_2	b_1	c_2	$\cdots$
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Now consider:

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Compute optimal repair. How?

For each $A = a_i$ compute optimal repair of $B \to C$, keep the largest.

Example 4: $\emptyset \to A$

$\emptyset \to A$

A	B	C	D
a_1	b_1	c_1	$\cdots$
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Compute optimal repair. How?

We keep a single value of A, namely the most frequent one.

Now consider:

$\emptyset \to A$
$B \to C$

Compute optimal repair. How?

For each $A = a_i$ compute optimal repair of $B \to C$, keep the largest.

Consensus rule: if Σ contains $\emptyset \to A$, then compute the optimal repair for each value $A = a_1, a_2 \ldots$, return the largest.

Example 5

$$A \to B$$
$$AC \to D$$

A	B	C	D
a_1	b_1	c_1	$\cdots$
a_1	b_2	c_1	$\cdots$
a_1	b_2	c_2	$\cdots$
a_2	b_1	c_1	$\cdots$
a_2	b_1	c_2	$\cdots$
a_2	b_2	c_3	$\cdots$
a_3	$\cdots$	$\cdots$	$\cdots$
	$\cdots$		

Compute optimal repair. How?

Example 5

$$A \to B$$
$$AC \to D$$

A	B	C	D
a_1	b_1	c_1	$\cdots$
a_1	b_2	c_1	$\cdots$
a_1	b_2	c_2	$\cdots$
a_2	b_1	c_1	$\cdots$
a_2	b_1	c_2	$\cdots$
a_2	b_2	c_3	$\cdots$
a_3	$\cdots$	$\cdots$	$\cdots$
	$\cdots$		

Compute optimal repair. How?

For each value $A = a_i$, compute the optimal repair of the residual:

$$\emptyset \to B$$
$$C \to D$$

Use the consensus rule.

Example 5

$A \to B$
$AC \to D$

A	B	C	D
a_1	b_1	c_1	$\cdots$
a_1	b_2	c_1	$\cdots$
a_1	b_2	c_2	$\cdots$
a_2	b_1	c_1	$\cdots$
a_2	b_1	c_2	$\cdots$
a_2	b_2	c_3	$\cdots$
a_3	$\cdots$	$\cdots$	$\cdots$
	$\cdots$		

For each value $A = a_i$, compute the optimal repair of the residual:

$\emptyset \to B$
$C \to D$

Use the consensus rule.

Compute optimal repair. How?

Common LHS rule: if all LHS contain A, $\Sigma = \{AX_1 \to Y_1, AX_2 \to Y_2, \ldots\}$, then repair separately each $A = a_i$.

Example 6

$A \to B$
$B \to A$

A	B	C	D
a_1	b_1	c_1	$\cdots$
a_1	b_2	c_1	$\cdots$
a_1	b_2	c_2	$\cdots$
a_2	b_1	c_1	$\cdots$
a_2	b_1	c_2	$\cdots$
a_2	b_2	c_3	$\cdots$
a_3	$\cdots$	$\cdots$	$\cdots$
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Compute optimal repair. How?

Example 6

$A \to B$
$B \to A$

A	B	C	D
a_1	b_1	c_1	$\cdots$
a_1	b_2	c_1	$\cdots$
a_1	b_2	c_2	$\cdots$
a_2	b_1	c_1	$\cdots$
a_2	b_1	c_2	$\cdots$
a_2	b_2	c_3	$\cdots$
a_3	$\cdots$	$\cdots$	$\cdots$
	$\cdots$		

Compute optimal repair. How?

Find a maximal matching the bipartite graph $(A, B, \Pi_{AB}(R))$.

A maximal matching in a bipartite graph can be found in PTIME using the "Hungarian Algorithm".

Last Example

$A \to B$
$B \to A$
$AB \to C$

A	B	C	D
a_1	b_1	c_1	$\cdots$
a_1	b_2	c_1	$\cdots$
a_1	b_2	c_2	$\cdots$
a_2	b_1	c_1	$\cdots$
a_2	b_1	c_2	$\cdots$
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Compute optimal repair. How?

Last Example

$A \to B$
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$AB \to C$

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a_2	b_1	c_2	$\cdots$
a_2	b_2	c_3	$\cdots$
a_3	$\cdots$	$\cdots$	$\cdots$
	$\cdots$		

Compute optimal repair. How?

For each pair $A = a_i, B = b_j$ compute optimal repair.

Weight of edge (a_i, b_j) is the size of the repair.

Find a maximal weighted matching in bipartite graph.

Last Example

$$A \to B$$
$$B \to A$$
$$AB \to C$$

A	B	C	D
a_1	b_1	c_1	$\cdots$
a_1	b_2	c_1	$\cdots$
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a_2	b_1	c_1	$\cdots$
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a_2	b_2	c_3	$\cdots$
a_3	$\cdots$	$\cdots$	$\cdots$
	$\cdots$		

Compute optimal repair. How?

Marriage Rule

For each pair $A = a_i, B = b_j$ compute optimal repair.

Weight of edge (a_i, b_j) is the size of the repair.

Find a maximal weighted matching in bipartite graph.

The Algorithm

[Livshits et al., 2020]
Given Σ, R, compute minimal repair that satisfies Σ.

- If $\Sigma = \emptyset$ then return R.
- Common LHS Rule If all LHS contain A, then repair each $A = a_i$. Return their union.

The Algorithm

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  Return the best repair.
- Marriage Rule If $\boldsymbol{U}^+ = \boldsymbol{V}^+$ and every rule has on the LHS either $\boldsymbol{U}$ or $\boldsymbol{V}$, then compute optimal repair for all pairs $\boldsymbol{U} = \boldsymbol{u}_i$, $\boldsymbol{V} = \boldsymbol{v}_j$
  Return maximal matching in weighted bipartite graph.

The Algorithm

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Given Σ, R, compute minimal repair that satisfies Σ.

- If $\Sigma = \emptyset$ then return R.
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- Marriage Rule If $\boldsymbol{U}^+ = \boldsymbol{V}^+$ and every rule has on the LHS either $\boldsymbol{U}$ or $\boldsymbol{V}$, then compute optimal repair for all pairs $\boldsymbol{U} = \boldsymbol{u}_i$, $\boldsymbol{V} = \boldsymbol{v}_j$ Return maximal matching in weighted bipartite graph.
- None of the above? Fail The problem is NP-hard.

Discussion

- Repairing for FDs: Dichotomy Theorem in [Livshits et al., 2020]. For each Σ, the the problem is either in PTIME or NP-hard.
- Data Exchange. Constraints are TGDs, LHS restricted to an input source database, RHS restricted to a target database. The repair is done via chase.
- A few other hardness results are known for repairing specific constraints (e.g. denial constraints).
- Related to the MAP problem in graphical models.

Incomplete Databases

Incomplete Databases

- A simple, pure theoretical concept that allows us to reason about different possible states of the database.
- Originally introduced by Imielinski and Lipski [Imielinski and Jr., 1984].
- I used these references: [Abiteboul et al., 1995, Chap.19], [Green and Tannen, 2006], [Libkin, 2014].

Definition

Recall: a database instance is $\boldsymbol{D} = (R_1^D, R_2^D, \ldots)$.

Let $\mathcal{N}$ be the set of all database instances.

Definition

An incomplete database is a set $\mathcal{I} \subseteq \mathcal{N}$.

Example all possible repairs of $\boldsymbol{D}$ w.r.t. Σ, $\mathcal{I} = \{\boldsymbol{D}_1, \boldsymbol{D}_2, \ldots\}$.

Possible Worlds, PWD.

Problems

How do we represent an incomplete database compactly?

How do we compute queries over incomplete databases?

Representation

Representations

- Codd tables.
- v-tables of naive tables.
- c-tables or conditional-tables. Special case:
  - ?-tables
  - or-tables

Representations

- Codd tables.
- v-tables of naive tables. We start here
- c-tables or conditional-tables. Special case:
  - ?-tables
  - or-tables

v-Tables

Dom = and infinite domain of values: $a, b, c, \ldots$

Null = an infinite set of marked NULLs: $\perp_1, \perp_2, \ldots$

v-Tables

Dom = and infinite domain of values: $a, b, c, \dots$
Null = an infinite set of marked NULLs: $\bot_1, \bot_2, \dots$

Definition

A v-table (a.k.a. naive table) is a finite set $R^I \subseteq (\text{Dom} \cup \text{Null})^k$.
Its semantics is: $[[R^I]] = \{\nu(R^I) \mid \nu : \text{Null} \rightarrow \text{Dom}\}$.

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Example $R^I =$

Name	City
Alice	$\perp_1$
Bob	SF
Carol	$\perp_2$
Dave	$\perp_1$

What is $[[R^I]]$?

v-Tables

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What is $[[R^I]]$?

Name	City
Alice	a
Bob	SF
Carol	a
Dave	a

Name	City
Alice	a
Bob	SF
Carol	b
Dave	a

Name	City
Alice	a
Bob	SF
Carol	c
Dave	a

...

Single restriction: Alice and Dave are in the same "City".

Codd Tables

Definition

A Codd table is a v-table where all marked nulls are distinct.

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Example $R^I =$

Name	City
Alice	$\perp_1$
Bob	SF
Carol	$\perp_2$
Dave	$\perp_3$

What is $[[R^I]]$?

Same as before, but now there is no restriction for Alice and Dave to be in the same city.

C-Tables

Definition

A C-table is a v-table where tuples are annotated with Boolean formulas, plus one global formula Φ.

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Example $R^I =$

Name	City	
Alice	$\perp_1$	X_1
Bob	SF	$X_1 \wedge (\perp_2 = \text{'SF'})$
Carol	$\perp_2$	$\texttt{true}$
Dave	$\perp_1$	X_2

$\Phi = X_1 \vee X_2$

C-Tables

Definition

A C-table is a v-table where tuples are annotated with Boolean formulas, plus one global formula Φ.

Example $R^I =$

Name	City	
Alice	$\bot_1$	X_1
Bob	SF	$X_1 \wedge (\bot_2 = \text{'SF'})$
Carol	$\bot_2$	$\texttt{true}$
Dave	$\bot_1$	X_2

$\Phi = X_1 \vee X_2$

Alice, Bob present only if $X_1 = \texttt{true}$.

Bob is present only if, in addition, Carol lives in SF

Dave is present only if $X_2 = \texttt{true}$.

Alice or Dave or both are present.

Special case of C-Tables: Maybe Tables

Definition

A maybe-table, or ?-table is a conventional table R^I where each tuple is annotated by a ?. Semantics: $[[R^I]] = \{R \mid R \subseteq R^I\}$.

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Example $R^I =$

Name	City	
Alice	Seattle	?
Bob	SF	?
Carol	Boston	?
Dave	Seattle	?

Semantics: $\mathcal{P}(R^I)$ (16 possible worlds).

This is a special of a c-table. Why?

Special case of C-Tables: OR-Table

Definition

An or-table is like a conventional table where each value can be an or-set.

An or-set, is a set whose meaning is "exactly one of its elements".
E.g. $\langle a, b, c\rangle$ means a or b or c.

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Example $R^I =$

What is $[[R^I]]$?

Name	City
Alice	⟨SF, Boston⟩
Bob	SF
Carol	Boston
Dave	⟨Seattle, SF⟩

Special case of C-Tables: OR-Table

Definition

An or-table is like a conventional table where each value can be an or-set.

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Example $R^I =$

Name	City
Alice	⟨SF, Boston⟩
Bob	SF
Carol	Boston
Dave	⟨Seattle, SF⟩

What is $[[R^I]]$?

Name	City
Alice	SF
Bob	SF
Carol	Boston
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Discussion

- Incomplete databases are a very general abstraction, meant to capture several scenarios:
  - Standard NULLs define an incomplete database.
  - Repairs for FDs can be described as an incomplete database.
  - Or-sets are a natural way to express alternatives.
- We saw incomplete *tables*; this extends to incomplete *databases*.
- We used the Closed World Assumption, CWA.
  Alternative: Open World Assumption, OWA.
- An incomplete database system: [Antova et al., 2007].

Queries on Incomplete Databases

Querying an Incomplete Database

Fix a query Q.

Definition

If $\mathcal{I} = \{\boldsymbol{D}_1, \boldsymbol{D}_2, \ldots\}$ is an incomplete database, then

$$Q(\mathcal{I}) \stackrel{\text{def}}{=} \{Q(\boldsymbol{D}_1), Q(\boldsymbol{D}_2), \ldots\}$$

How do we represent $Q(\mathcal{I})$?

Closed Representation System

Fix a representation system $\mathcal{R}$ (e.g. v-tables)
and a query language $\mathcal{L}$ (e.g. CQ or FO).

Definition

$\mathcal{R}$ is closed under $\mathcal{L}$, if for any $\boldsymbol{D}^I \in \mathcal{R}$ and any query $Q \in \mathcal{L}$, there exists a representation A^I for the query answer, in other words $[[A^I]] = Q([[\boldsymbol{D}^I]])$.

Closed Representation Systems

Fact

V-tables are not closed under FO:

Proof $Q(X) = R(X) \wedge \neg S(X)$, $R = \{1, 2\}$, $S^I = \{\perp\}$.
Then $Q([[R, S^I]]) = \{\{1, 2\}, \{1\}, \{2\}\}$; not representable as a v-table.

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Then $Q([[R, S^I]]) = \{\{1,2\},\{1\},\{2\}\}$; not representable as a v-table.

Theorem

C-tables are closed under FO.

Discussion

Computing and representing all possible answers $Q(\mathcal{I})$ is difficult, and often not very informative.

A better alternative: certain answers

Also an option (but less desirable): possible answers

Certain Answers and Possible Answers

Definition

A *certain tuple* is a tuple t s.t. $\forall \boldsymbol{D} \in \mathcal{I}$, $t \in Q(\boldsymbol{D})$. Their set: $\texttt{cert}(Q, \mathcal{I})$
A *possible tuple* is a tuple t s.t. $\exists \boldsymbol{D} \in \mathcal{I}$, $t \in Q(\boldsymbol{D})$. Their set: $\texttt{poss}(Q, \mathcal{I})$

Equivalently:

$$\texttt{cert}(Q, \mathcal{I}) = \bigcap \{Q(\boldsymbol{D}) \mid \boldsymbol{D} \in \mathcal{I}\}$$

$$\texttt{poss}(Q, \mathcal{I}) = \bigcup \{Q(\boldsymbol{D}) \mid \boldsymbol{D} \in \mathcal{I}\}$$

Example

Querying v-tables:

$$R^I = \begin{array}{|c|c|} \hline x & \perp_1 \\ y & \perp_1 \\ z & \perp_2 \\ \hline \end{array} \qquad S^I = \begin{array}{|c|c|} \hline \perp_1 & a \\ \perp_2 & b \\ \perp_2 & c \\ \perp_3 & d \\ \hline \end{array}$$

$Q(X, Z) = R(X, Y) \wedge S(Y, Z)$

What are the certain tuples? The possible tuples?

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Querying v-tables:

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$Q(X,Z) = R(X,Y) \wedge S(Y,Z)$

What are the certain tuples? The possible tuples?

$\texttt{cert}(Q,\mathcal{I}) =$

x	a
y	a
z	b
z	c

$\texttt{poss}(Q,\mathcal{I}) =$

x	a
x	b
$\ldots$	
z	d

The cartesian product.

Strong/Weak Representation Systems

Following [Libkin, 2014].

Fix a representation system $\mathcal{R}$, query language $\mathcal{L}$.

$\mathcal{R}$ is a strong representation system for $\mathcal{L}$ if it is closed under $\mathcal{L}$, i.e. for all $\boldsymbol{D}^I \in \mathcal{R}, Q \in \mathcal{L}$, $\exists A^I \in \mathcal{R}$ such that:

$$[[A^I]] = \{Q(\boldsymbol{D}) \mid \boldsymbol{D} \in [[\boldsymbol{D}^I]]\}$$

$\mathcal{R}$ is a weak representation system for $\mathcal{L}$ if for all $\boldsymbol{D}^I \in \mathcal{R}, Q \in \mathcal{L}$, $\exists A^I \in \mathcal{R}$ such that, for all $q \in \mathcal{L}$

$$\texttt{cert}(q, [[A^I]]) = \texttt{cert}(q, \{Q(\boldsymbol{D}) \mid \boldsymbol{D} \in [[\boldsymbol{D}^I]]\})$$

In other words, we cannot represent the possible answers exactly, but we can represent all the certain answers on all future queries q.

V-Tables are a Weak Representation System for UCQs

Theorem

V-tables are a weak representation system for UCQs.

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V-tables are a weak representation system for UCQs.

$R^I =$

x	$\perp_1$
y	$\perp_1$
z	$\perp_2$

$S^I =$

$\perp_1$	a
$\perp_2$	b
$\perp_2$	c
$\perp_3$	d

$$Q(X, Y, Z) = R(X, Y) \wedge S(Y, Z)$$

V-Tables are a Weak Representation System for UCQs

Theorem

V-tables are a weak representation system for UCQs.

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y	$\perp_1$
z	$\perp_2$

$S^I =$

$\perp_1$	a
$\perp_2$	b
$\perp_2$	c
$\perp_3$	d

$Q(R^I, S^I) =$

x	$\perp_1$	a
y	$\perp_1$	a
z	$\perp_2$	b
z	$\perp_2$	c

$$Q(X, Y, Z) = R(X, Y) \wedge S(Y, Z)$$

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- Does SQL adopt the possible world semantics of Codd tables?

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- **Theorem** when Q is in FO, then the complexity of $\texttt{cert}(Q, \boldsymbol{D}^I)$ where $\boldsymbol{D}^I$ is a v-database is co-NP hard.

Announcement

- No lectures next week! Join the workshop at Simons.
- The following week: two guest lectures by Val Tannen on semirings and their applications to databases.

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